R&D Productivity And The Nexus Between Product Substitutability And Innovation: Theory And Experimental Evidence

Appendix

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A A Microfoundation of the Theoretical Model

Here, we provide a microfoundation for the reduced-form model we outlined in the main text. Our model has two periods. In the first period, the duopolists undertake R&D investment, while in the second period, production takes place in a linear duopoly under product substitutability.

A.1 The Imperfect Product Substitutability Cournot Stage Game

In the second period, firms set their output level and, subsequently, prices adjust so that demand equals supply. Each firm i has a marginal production cost κ_i and faces demand described by the inverse demand function

$$p_i = b_i - \theta q_j - \gamma q_i,$$

where p_i denotes the price, q_i denotes the output of firm i, b_i represents the consumers' highest willingness to pay for the product of firm i (will be referred to as a *quality* parameter), q_j is the rival's output, and θ is a parameter that captures the substitutability of the two goods in the preferences of the consumers. We assume $\gamma > 0$ and $b_i > 0$, $\gamma > \theta \geq 0$.[1]

Denote $n_i \equiv b_i - \kappa_i$ and refer to it as *cost-adjusted quality.* To fix ideas, let us treat firm $i = 1$ to be the one that is (weakly) more *technologicaly advantaged*, i.e. $n_1 \geq n_2$. Assume that

$$b_i > \kappa_i, \tag{1}$$

which ensures that $n_i > 0$ for all i.

Firms choose simultaneously and independently their output level to maximize their second-period profits. We refer to these profits as the duopolists' economic rents. Firm i's best-response function is

$$q_i = max\{\frac{b_i - \kappa_i - \theta q_j}{2\gamma}, 0\}, \quad i = 1, 2. \tag{2}$$

Denote with star the (pure strategy) Nash equilibrium values. The Nash equilibrium is given by the solution to the system of the above best-response functions. The equilibrium

[1] This environment emerges from price-taking maximization of a linear-quadratic utility function of a numeraire good and the goods produced by the two firms in question, together with the assumption that whenever a firm's good faces zero demand, the price that clears the market is the lowest price from all possible prices that are compatible with zero demand for this good.

economic rents attained by duopolist i equal

$$\Pi_i^* = (b_i - \theta q_j^* - \gamma q_i^* - \kappa_i) q_i^* = \gamma \left(q_i^*\right)^2$$

where the second equality follows from using the best-response function of firm i. It follows that if $\theta \leq 2\gamma \frac{n_2}{n_1}$, then, the Nash equilibrium is given by $q_i = q_i^*$, where

$$q_i^* = \frac{2\gamma(b_i - \kappa_i) - \theta(b_j - \kappa_j)}{4\gamma^2 - \theta^2},$$

with $q_1^* - q_2^* = \frac{(2\gamma+\theta)(n_1-n_2)}{4\gamma^2-\theta^2} \geq 0$. If, on the other hand, $2\gamma \frac{n_2}{n_1} < \theta < \gamma$ (and hence $n_1 > n_2$), then, the Nash equilibrium is given by $q_2^* = 0, q_1^* = \frac{n_1}{2\gamma} > \frac{n_2}{\theta}$, where the less (technologically) advantaged firm is at a corner solution. In any case, the more advantaged firm earns higher economic rents.

A.2 R&D and Cost-Adjusted Qualities

Observe from above that both output levels (and hence economic rents) depend on both n_1 and n_2.

We postulate that the difference between n_1 and n_2 depends on the outcome of the non-cooperative R&D investment of the firms in the first stage of their interaction.

Specifically, we assume that

$$n_i = n + k_i,$$

for all $i = 1, 2$, where $n > 1$ is an exogenous parameter which fixes, in effect, the average cost-adjusted quality in the industry, and

$$k \in \{-1, 0, 1\},$$

where $k_i = 0$ for all $i = 1, 2$ in a levelled (in the second stage) industry, whereas for the case of an unlevelled (in the second stage) industry, $k_1 = 1$ and $k_2 = -1$. We therefore have that in a levelled industry $n_1 = n_2 = n$, while in an unlevelled industry $n_1 = n+1 > n > n-1 = n_2$.

A.3 Investing for Improvements of Cost-Adjusted Quality

In the first period, firms invest towards gaining an advantage in terms of their cost-adjusted quality (and, thereby, economic rents) vis-a-vis their competitor. In our setup, as described above, to gain such an advantage a firm needs to improve the cost-adjusted quality by one level so that, depending on the R&D outcomes, either catches up with the competitor or becomes the leader.

The probability of improving the cost-adjusted quality by one unit (referred to as "research capacity") is set to be $p_i = f(a_i)$, where $f(\cdot)$ is an increasing and (weakly) concave function of investment $a_i \in [0, \overline{a}]$ with $f(0) = 0$ and $f(\overline{a}) = 1$. Let also the cost of R&D investment be given by a function $\widehat{C}(a_i, K)$, such that $\widehat{C}(\cdot, K)$ is increasing and convex and $\widehat{C}(a_i, \cdot)$ is decreasing for any a_i with $\widehat{C}(0, K) = 0$ and $\lim_{a\to\overline{a}} \widehat{C}(a, K) = \infty$, for any K.

Observe that in order to implement a research capacity p_i, firm i needs to invest $a_i = f'^{-1}(p_i)$. We thus have that the cost from implementing a research capacity p_i is

$$C(p_i, K) = \widehat{C}(f'^{-1}(p_i), K).$$

As can easily be verified, this function is such that $C(., K)$ is increasing and convex and $C(a_i, \cdot)$ is decreasing for any a_i, with $C(0, K) = 0$ and $\lim_{p\to 1} C(p, K) = \infty$, for any K, as it is assumed in the reduced-form model.

Finally, note that using $p_i = a_i/100$ and $\widehat{C}(a_i, K) = \frac{1}{K}\frac{a_i}{100-a_i}$ gives the cost function $C(p_i, K)$ used in the experiments.

A.4 Product Substitutability and Economic Rents

Recall from the discussion of the Cournot duopoly earlier that if $\theta \leq 2\gamma\frac{n-1}{n+1}$, then, firm i earns economic rents

$$\Pi_i^* = \gamma\left[\frac{2\gamma n_i - \theta n_j}{4\gamma^2 - \theta^2}\right]^2.$$

Therefore, in a neck-and-neck industry, where $n_1 = n_2 = n$, we have that each firm earns economic rents

$$\Pi_0^* \equiv \gamma\left[\frac{n}{2\gamma + \theta}\right]^2.$$

Moreover, in an unlevelled industry, where $n_1 > n > n_2$, we have

$$\Pi_1^* = \gamma\left[\frac{2\gamma n_1 - \theta n_2}{4\gamma^2 - \theta^2}\right]^2$$

and

$$\Pi_2^* = \gamma \left[\frac{2\gamma n_2 - \theta n_1}{4\gamma^2 - \theta^2} \right]^2.$$

In terms of the notation used in the reduced-form model, we have $\pi_h = \Pi_1^*, \pi_l = \Pi_2^*$ and $\pi_s = \Pi_0^*$. Below, we derive the conditions on γ, θ, n that will ensure that these economic rents satisfy the required properties in our reduced-form model.[2]

A.4.1 Meeting Conditions on Economic Rents

Given that non-cooperative economic rents are (as we have seen earlier) equal to γ times output squared, we have that the required conditions

$$\Pi_2^* < \Pi_0^*$$

$$\Pi_1^* > \Pi_0^*$$
$$\frac{\partial[\Pi_1^* - \Pi_0^*]}{\partial\theta} > 0$$
$$\frac{\partial[\Pi_0^* - \Pi_2^*]}{\partial\theta} < 0$$

are equivalent to

$$\left(\frac{2\gamma n_2 - \theta n_1}{4\gamma^2 - \theta^2} \right)^2 < \left(\frac{(2\gamma - \theta)n}{4\gamma^2 - \theta^2} \right)^2$$

$$\left(\frac{2\gamma n_1 - \theta n_2}{4\gamma^2 - \theta^2} \right)^2 > \left(\frac{(2\gamma - \theta)n}{4\gamma^2 - \theta^2} \right)^2$$

$$\frac{\partial[\left(\frac{2\gamma n_1 - \theta n_2}{4\gamma^2 - \theta^2} \right)^2 - \left(\frac{(2\gamma - \theta)n}{4\gamma^2 - \theta^2} \right)^2]}{\partial\theta} > 0$$

$$\frac{\partial[\left(\frac{(2\gamma - \theta)n}{4\gamma^2 - \theta^2} \right)^2 - \left(\frac{2\gamma n_2 - \theta n_1}{4\gamma^2 - \theta^2} \right)^2]}{\partial\theta} < 0.$$

Using that

$$\frac{2\gamma n_1 - \theta n_2}{4\gamma^2 - \theta^2} = \frac{(2\gamma + \theta) + (2\gamma - \theta)n}{4\gamma^2 - \theta^2}$$

[2] One can easily see that the required properties in our reduced-form model are also satisfied by the economics rents when $\theta > 2\gamma\frac{n_1}{n_2}$. The reason is that Π_0^* is clearly decreasing in θ and, in this environment, we have $\Pi_2^* = 0$ and $\Pi_1^* = \frac{n_1}{2\gamma}$, which is independent of θ.

$$\frac{2\gamma n_2 - \theta n_1}{4\gamma^2 - \theta^2} = \frac{-(2\gamma+\theta) + (2\gamma - \theta)n}{4\gamma^2 - \theta^2}$$

we have that we need γ, θ, n to be such that

$$\left(\frac{(2\gamma-\theta)n}{4\gamma^2-\theta^2}\right)^2 - \left(\frac{2\gamma n_2 - \theta n_1}{4\gamma^2-\theta^2}\right)^2 = [\frac{n}{2\gamma+\theta} - (\frac{n}{2\gamma+\theta} - \frac{1}{2\gamma-\theta})][\frac{n}{2\gamma+\theta} + (\frac{n}{2\gamma+\theta} - \frac{1}{2\gamma-\theta})] =$$

$$\frac{1}{2\gamma-\theta}[\frac{2n}{2\gamma+\theta} - \frac{1}{2\gamma-\theta}] > 0$$

$$\frac{2\gamma n_1 - \theta n_2}{4\gamma^2-\theta^2} - \frac{(2\gamma-\theta)n}{4\gamma^2-\theta^2} = \frac{1}{2\gamma-\theta} > 0$$

$$\frac{\partial[\left(\frac{2\gamma n_1-\theta n_2}{4\gamma^2-\theta^2}\right)^2 - \left(\frac{(2\gamma-\theta)n}{4\gamma^2-\theta^2}\right)^2]}{\partial\theta} = \frac{\partial[\left(\frac{1}{2\gamma-\theta}\right)^2 + 2\left(\frac{1}{2\gamma-\theta}\right)\left(\frac{n}{2\gamma+\theta}\right)]}{\partial\theta} = \frac{\partial[\left(\frac{1}{2\gamma-\theta}\right)^2 + 2\left(\frac{n}{(2\gamma)^2-\theta^2}\right)]}{\partial\theta} > 0$$

$$\frac{\partial[\left(\frac{(2\gamma-\theta)n}{4\gamma^2-\theta^2}\right)^2 - \left(\frac{2\gamma n_2-\theta n_1}{4\gamma^2-\theta^2}\right)^2]}{\partial\theta} = \frac{\partial[-\left(\frac{1}{2\gamma-\theta}\right)^2 + 2\left(\frac{1}{2\gamma-\theta}\right)\left(\frac{n}{2\gamma+\theta}\right)]}{\partial\theta} = \frac{\partial[-\left(\frac{1}{2\gamma-\theta}\right)^2 + 2\left(\frac{n}{(2\gamma)^2-\theta^2}\right)]}{\partial\theta} < 0.$$

It follows directly that $\pi_h^* > \pi_l^*$ and $\frac{\partial[\pi_h^* - \pi_l^*]}{\partial\theta} > 0$ are satisfied for any $\gamma > 0, \theta < \gamma, n > 0$. Turning to the remaining conditions, we clearly have that $\Pi_2^* < \Pi_0^*$ if and only if

$$2(2\gamma - \theta)n > 2\gamma + \theta,$$

which can be re-written as

$$\frac{\gamma}{\theta} > \frac{(1+2n)}{2(2n-1)}. \tag{3}$$

Clearly, given $n > 1, \gamma > \theta$ the above is satisfied if $1 + 2n \leq 2(2n-1)$; that is, if $n \geq 3/2$, which is feasible. To derive a condition that ensures that $\frac{\partial[\Pi_0^* - \Pi_2^*]}{\partial\theta} < 0$, note that

$$\frac{\partial[-\left(\frac{1}{2\gamma-\theta}\right)^2 + 2\left(\frac{n}{(2\gamma)^2-\theta^2}\right)]}{\partial\theta} = -2\left(\frac{1}{2\gamma-\theta}\right)^3 + 4n\theta\left(\frac{1}{(2\gamma)^2-\theta^2}\right)^2 =$$

$$-2\left(\frac{1}{2\gamma-\theta}\right)^2\left\{\frac{1}{2\gamma-\theta} - 2n\theta\left(\frac{1}{2\gamma+\theta}\right)^2\right\}.$$

This is negative if $\theta = 0$. For when $\theta > 0$, we need

$$n < \frac{(2\gamma + \theta)^2}{2\theta\,(2\gamma - \theta)}. \tag{4}$$

Note that the right-hand side goes to infinity as $\theta \to 0^+$, while it equals $9/2$ when $\theta \to \gamma^-$. Therefore, by continuity, there is a range of values for $0 < \theta < \gamma$ for which the above condition is satisfied for any given $n \geq 3/2$ (which ensures the previous condition).

Finally, notice that $\Pi_1^* - \Pi_0^* \geq \Pi_0^* - \Pi_2^*$, in which case (6) in the main text holds for any convex cost function, if

$$\left(\frac{2\gamma n_1 - \theta n_2}{4\gamma^2 - \theta^2}\right)^2 - \left(\frac{(2\gamma - \theta)n}{4\gamma^2 - \theta^2}\right)^2 \geq$$

$$\left(\frac{(2\gamma - \theta)n}{4\gamma^2 - \theta^2}\right)^2 - \left(\frac{2\gamma n_2 - \theta n_1}{4\gamma^2 - \theta^2}\right)^2.$$

This can be rewritten as

$$\frac{1}{2\gamma - \theta}[\frac{2n}{2\gamma + \theta} + \frac{1}{2\gamma - \theta}] \geq$$

$$\frac{1}{2\gamma - \theta}[\frac{2n}{2\gamma + \theta} - \frac{1}{2\gamma - \theta}]$$

which is clearly true for $\gamma > \theta$.

For our experiments, we choose the values $n = 2$ and $\gamma = 1$. For these values, and $\theta \in \{0.1, 0.2, 0.5, 0.6\}$, we have that $n > 3/2$ and equation (4) is satisfied, and π_h, π_l, π_s are as in Table 1 in the main text.

B Arbitrary Number of Steps in the Technology Ladder

Note that the strategic environment in a levelled industry is independent of the number of steps; thus, the analysis and results stay the same. In what follows, we restrict attention to an unlevelled industry. We remind the reader here that in an unlevelled industry, we do not find any novel results when adding R&D productivity to the nexus between competition and innovation.

Let $\sigma = 1, ..., S$ denote the technology gap (i.e. the number of the steps in the technology ladder) between the leader and the laggard, where $S \geq 1$ is a natural number (finite or infinite). Let $\pi_l(\sigma)$ be the rents of the laggard when the technology gap with the leader is equal to σ steps, and $\pi_h(\sigma)$ be the rents of the leader when the technology gap with the laggard is equal to σ steps, with $\sigma = 1, ..., S$. Let us also use the convention $\pi_l(S+1) = \pi_l(S)$ and $\pi_h(S+1) = \pi_h(S)$, and define $\pi_s \equiv \pi_l(0) \equiv \pi_h(0)$. As in our basic model, $\pi_l(\sigma)$, $\pi_h(\sigma)$ and π_s are functions of θ, and we refrain from writing explicitly this dependence to simplify notation.

We assume that $\pi_l(\cdot)$ is a (weakly) decreasing and convex function, whereas $\pi_l(\cdot)$ is a (weakly) increasing and concave function; that is:

$$0 \leq \pi_l(\sigma) - \pi_l(\sigma + 1) \leq \pi_l(\sigma - 1) - \pi_l(\sigma)$$

and

$$\pi_h(\sigma) - \pi_h(\sigma - 1) \geq \pi_h(\sigma + 1) - \pi_h(\sigma) \geq 0,$$

for all $\sigma = 2, .., S - 1$.

The remaining assumptions are the direct analogues of the ones in our basic model (where $\pi_l(\sigma + 1) = \pi_l(1) \equiv \pi_l$ and $\pi_h(\sigma + 1) = \pi_h(1) \equiv \pi_h$ for all $\sigma = 1, ..., S$). In particular, we assume that $\pi_h(S)$ is finite, and that

$$\pi_l(1) < \pi_s < \pi_h(1),$$

and so $\pi_l(\sigma) < \pi_s < \pi_h(\sigma)$ for all $\sigma = 1, ..., S$.

Finally, we assume that

$$\frac{\partial[\pi_h(\sigma) - \pi_h(\sigma - 1)]}{\partial\theta} > 0 \tag{5}$$

and

$$\frac{\partial[\pi_l(\sigma-1)-\pi_l(\sigma)]}{\partial\theta}<0 \tag{6}$$

for all $\sigma = 1, .., S$; that is, the function $\pi_h(\cdot)$ becomes steeper and the difference $\pi_h(1) - \pi_s$ becomes larger, while the function $\pi_l(\cdot)$ becomes flatter and the difference $\pi_s - \pi_l(1)$ becomes smaller as θ increases. It turns out that with these assumptions the main message of our analysis of the unlevelled industry is still valid: more competition causes the laggard to reduce R&D investment independently of the level of R&D productivity. We show this next.

To fix ideas, suppose that firm $i = 2$ is the laggard and that firm $i = 1$ is the leader with a technology gap of $\sigma = 1, ..., S$ steps at the time of R&D investment. The investment problem of the laggard is to maximize with respect to p_2:

$$(1-p_2)\left[p_1^*\pi_l(\sigma+1)+(1-p_1^*)\pi_l(\sigma)\right]+p_2\left[p_1^*\pi_l(\sigma)+(1-p_1^*)\pi_l(\sigma-1)\right]-C(p_2,K)=$$

$$[p_1^*\pi_l(\sigma+1)+(1-p_1^*)\pi_l(\sigma)]+$$
$$p_2\{p_1^*[\pi_l(\sigma)-\pi_l(\sigma+1)]+(1-p_1^*)[\pi_l(\sigma-1)-\pi_l(\sigma)]\}-\frac{c(p_2)}{K}.$$

Taking the first-order condition with respect to p_2, we have at an interior solution (i.e. when $p_2^* > 0$) that:

$$K\{p_1^*[\pi_l(\sigma)-\pi_l(\sigma+1)]+(1-p_1^*)[\pi_l(\sigma-1)-\pi_l(\sigma)]\}=c'(p_2^*). \tag{7}$$

Note that in the case of zero investment by the laggard (i.e. when $p_2^* = 0$), the equality is replaced with a "lower than or equal to" inequality.

The problem of the leader, in turn, is to maximize with respect to p_1:

$$(1-p_1)[p_2^*\pi_h(\sigma-1)+(1-p_2^*)\pi_h(\sigma)]+p_1[p_2^*\pi_h(\sigma)+(1-p_2^*)\pi_h(\sigma+1)]-C(p_1,K)=$$

$$[p_2^*\pi_h(\sigma-1)+(1-p_2^*)\pi_h(\sigma)]+$$
$$p_1\{p_2^*[\pi_h(\sigma)-\pi_h(\sigma-1)]+(1-p_2^*)[\pi_h(\sigma+1)-\pi_h(\sigma)]\}-\frac{c(p_1)}{K}.$$

Taking the first-order condition with respect to p_1, we have at an interior solution (i.e. when $p_1^* > 0$) that

$$K\{p_2^*[\pi_h(\sigma)-\pi_h(\sigma-1)]+(1-p_2^*)[\pi_h(\sigma+1)-\pi_h(\sigma)]\}=c'(p_1^*). \tag{8}$$

Note that in the case of zero investment by the leader (i.e. when $p_1^* = 0$), the equality is replaced with a "lower than or equal to" inequality.

As with the basic model, hereafter, we restrict attention to the (more interesting) case where there is strictly positive investment from the laggard in the equilibrium. To start with, we observe that if $p_1^* = 0$, then, the laggard's R&D investment decreases with product substitutability independently of R&D productivity. To see this, note that the above first-order condition of the laggard becomes

$$K[\pi_l(\sigma - 1) - \pi_l(\sigma)] = c'(p_2^*)$$

and observe that the left-hand side is decreasing in θ given our assumptions above.

Turning to the case where both firms invest, to find the effect of θ on the equilibrium research capacities, we need to use the Implicit Function Theorem. So, dropping the asterisks for notational simplicity, and using the two first-order conditions of the laggard and the leader (7) and (8), we have that

$$\begin{bmatrix} -c''(p_1) & K\{[\pi_h(\sigma) - \pi_h(\sigma - 1)] - [\pi_h(\sigma + 1) - \pi_h(\sigma)]\} \\ K\{[\pi_l(\sigma) - \pi_l(\sigma + 1)] - [\pi_l(\sigma - 1) - \pi_l(\sigma)]\} & -c''(p_2) \end{bmatrix} \times$$

$$\begin{bmatrix} \partial p_1/\partial\theta \\ \partial p_2/\partial\theta \end{bmatrix} = \begin{bmatrix} -K\{p_2\frac{\partial[\pi_h(\sigma)-\pi_h(\sigma-1)]}{\partial\theta} + (1-p_2)\frac{\partial[\pi_h(\sigma+1)-\pi_h(\sigma)]}{\partial\theta}\} \\ -K\{p_1\frac{\partial[\pi_l(\sigma)-\pi_l(\sigma+1)]}{\partial\theta} + (1-p_1)\frac{\partial[\pi_l(\sigma-1)-\pi_l(\sigma)]}{\partial\theta}\} \end{bmatrix}.$$

Therefore, the effect of θ on laggard's R&D investment is determined by the sign of

$$\partial p_2/\partial\theta =$$

$$\frac{c''(p_1)K\{p_1\frac{\partial[\pi_l(\sigma)-\pi_l(\sigma+1)]}{\partial\theta} + (1-p_1)\frac{\partial[\pi_l(\sigma-1)-\pi_l(\sigma)]}{\partial\theta}\}}{c''(p_1)c''(p_2) - K\{[\pi_h(\sigma) - \pi_h(\sigma-1)] - [\pi_h(\sigma+1) - \pi_h(\sigma)]\}K\{[\pi_l(\sigma) - \pi_l(\sigma+1)] - [\pi_l(\sigma-1) - \pi_l(\sigma)]\}}$$

$$\frac{K\{[\pi_l(\sigma) - \pi_l(\sigma+1)] - [\pi_l(\sigma-1) - \pi_l(\sigma)]\}K\{p_2\frac{\partial[\pi_h(\sigma)-\pi_h(\sigma-1)]}{\partial\theta} + (1-p_2)\frac{\partial[\pi_h(\sigma+1)-\pi_h(\sigma)]}{\partial\theta}\}}{c''(p_1)c''(p_2) - K\{[\pi_h(\sigma) - \pi_h(\sigma-1)] - [\pi_h(\sigma+1) - \pi_h(\sigma)]\}K\{[\pi_l(\sigma) - \pi_l(\sigma+1)] - [\pi_l(\sigma-1) - \pi_l(\sigma)]\}}.$$

Note that the common denominator is positive by the convexity of the cost function and $\pi_l(\cdot)$ and the concavity of $\pi_l(\cdot)$. In addition, both numerators are negative by the convexity of the cost function and $\pi_l(\cdot)$, and assumptions (5) and (6). Therefore, the laggard's R&D investment decreases with product substitutability in this case as well.

C Deterministic Innovation

Here, we study the implications of deterministic innovation in our ladder-type model. In such an environment, the "probability of success" (or "research capacity") is either 0 or 1, and so firms' R&D problem becomes a discrete-choice problem. As a result, equilibrium research capacities will be either flat or (non-trivial) step functions over the whole range of θs. Therefore, the shape of these will in general be different from the shape of equilibrium research capacities under stochastic innovation, which is smooth and, over some θs, monotone. Let $c \equiv c(1)$ and assume that indifferences are resolved in favour of innovating (i.e. of setting research capacity to one). Denote with $\mathbf{1}\{\Omega\}$ the index function that takes value one if Ω holds and value zero otherwise.

C.1 The Investment Problem in an Unlevelled Industry

C.1.1 The (First-Stage) Laggard's Problem

The laggard maximizes with respect to $p_2 \in \{0, 1\}$ its expected profits

$$\pi_l + p_2(1 - p_1^*)(\pi_s - \pi_l) - c(p_2)/K,$$

where $p_1^* \in \{0, 1\}$. We have that the laggard decides to innovate (i.e. to set $p_2 = 1$) if

$$K(1 - p_1^*)(\pi_s - \pi_l) \geq c;$$

that is,

$$p_2^* = \mathbf{1}\{K(1 - p_1^*)(\pi_s - \pi_l) \geq c\}.$$

C.1.2 The (First-Stage) Leader's Problem

The leader maximizes with respect to $p_1 \in \{0, 1\}$ its expected profits

$$\pi_h - (1 - p_1)p_2^*(\pi_h - \pi_s) - c(p_1)/K,$$

where $p_2^* \in \{0, 1\}$. We have that the leader decides to innovate (i.e. to set $p_1 = 1$) if

$$Kp_2^*(\pi_h - \pi_s) \geq c;$$

that is,

$$p_1^* = \mathbf{1}\{Kp_2^*(\pi_h - \pi_s) \geq c\}.$$

C.1.3 Equilibrium Investment

Observe that, in equilibrium, it cannot be the case that $\{p_1^*, p_2^*\} = \{1, 0\}$ because if $p_2^* = 0$, then, the leader's benefit from innovating is zero, and so the equilibrium would also set the research capacity to zero. Related, it cannot be the case, in equilibrium, that $\{p_1^*, p_2^*\} = \{1, 1\}$ because if $p_1^* = 1$, then, the laggard's benefit from innovating is zero, and so equilibrium would set the research capacity to zero. The above optimality conditions thus imply the following two cases in equilibrium.

First, $\{p_1^*, p_2^*\} = \{0, 0\}$ if $K(\pi_s - \pi_l) < c$. Second, $\{p_1^*, p_2^*\} = \{0, 1\}$ if $K(\pi_s - \pi_l) \geq c > K(\pi_h - \pi_s)$. Therefore, in the model with deterministic innovation, the leader never innovates. Consequently, the laggard's innovation decision is given by:

$$p_2^* = \mathbf{1}\{K(\pi_s - \pi_l) \geq c\}.$$

Turning to the impact of a higher degree of product substitutability on the laggard's investment, we clearly have, given our assumptions (2) and (3) in the main text, that an increase in θ (weakly) decreases p_2^*, which echoes our result in the main text under stochastic innovation, where the laggard's investment is decreasing in θ.

C.2 The Investment Problem in a Levelled Industry

Each firm $i = 1, 2$, while taking as given the opponent's choice, p_{-i}, maximizes with respect to $p_i \in \{0, 1\}$ the following expected profits

$$p_{-i}^*\pi_l + (1 - p_{-i}^*)\pi_s + p_i[p_{-i}^*(\pi_s - \pi_l) + (1 - p_{-i}^*)(\pi_h - \pi_s)] - \frac{c(p_i)}{K}.$$

At optimum, we have:

$$p_i = \mathbf{1}\{K[p_{-i}^*(\pi_s - \pi_l) + (1 - p_{-i}^*)(\pi_h - \pi_s)] \geq c\}.$$

Therefore, in a symmetric equilibrium, where $p_i^* = p_{-i}^* = p^*$, we have that

$$p^* = \mathbf{1}\{K[p^*(\pi_s - \pi_l) + (1 - p^*)(\pi_h - \pi_s)] \geq c\}.$$

Suppose that $p^* = 1$. Then, the above equilibrium condition implies that $K(\pi_s - \pi_l) \geq c$ must be true. If, on the other hand, $p^* = 0$, then, the above equilibrium condition implies that $K(\pi_h - \pi_s) < c$ must be true. Thus observe that if $(\pi_s - \pi_l) < c/K \leq (\pi_h - \pi_s)$, then, there is no symmetric equilibrium. Note also that if $(\pi_h - \pi_s) < c/K \leq (\pi_s - \pi_l)$, then, we have multiplicity of equilibria: one equilibrium features innovation and the other equilibrium features no innovation. These are in contrast to what we find in our model with stochastic innovation.

As in our model with stochastic innovation, we restrict attention to an environment where there exists a unique symmetric equilibrium for all values of θs. We thus assume hereafter that either $max\{(\pi_h - \pi_s), (\pi_s - \pi_l)\} < c/K$ for all θ, or $min\{(\pi_h - \pi_s), (\pi_s - \pi_l)\} \geq c/K$ for all θ. In the former case, there is only one symmetric equilibrium where firms do not innovate for all θ, while in the latter case, there is only one symmetric equilibrium where firms do innovate for all θs. Thus, in both cases the research capacity is *constant* over all θs. These findings are consistent with our result in the main text under stochastic innovation, where the symmetric equilibrium investment might not be an increasing function of θ.

D Continuous Innovation

Here, we study the R&D problem of Cournot duopolists who choose at the first stage of their interaction, simultaneously and independently, an investment $a_i \in [0, \overline{a}]$, $i = 1, 2$ that costs $\frac{\widehat{c}(a_i)}{K}$ and determines the maximum possible increase in their *cost-adjusted quality.* We allow for the presence of a post-investment shock, $x_i \in [0, \overline{x})$, which may reduce the impact of the investment. Specifically, we postulate that the realized increase in the cost-adjusted quality (i.e. innovation) is equal to $a_i x_i$.

After both innovations are realized, production takes place at the second stage of the firms' interaction, in a linear Cournot duopoly under product substitutability with constant marginal cost κ_i and linear demand $p_i = b_i - \gamma q_i - \theta q_{-i}$, $0 < \theta < \gamma$, where the second-stage cost-adjusted quality of firm i is defined by $n_i \equiv b_i - \kappa_i > 0$, for any $-i \neq 1$, $i = 1, 2$. As is our convention so far, denote hereafter with $i = 1$ the leader in the industry and with $i = 2$ the laggard at the *second* stage, i.e. $n_1 \geq n_2$.

Denote with $G(x_i)$ the cdf of the shock x_i, $i = 1, 2$. The firms' shocks are i.i.d. Let $\widehat{x}$ be the mean shock faced by firms. The case of deterministic innovation (i.e. when there is no uncertainty) is captured by the case where G is the Dirac distribution, $G^D(x)$, putting all probability at $x = \overline{x}$ (and hence $\widehat{x} = \overline{x}$) with $\overline{x} = 1$.

Let the cost-adjusted qualities of the leader and the laggard at the first stage be equal to $n+\sigma$ and $n-\sigma$, respectively, where, with some abuse of notation in this Section, $\sigma = 1, ..., S$, $S < n$; that is, 2σ is the inherited "technology gap" between the leader and the laggard at the first stage (i.e. before investment taking place). The case with $\sigma = 0$ captures a levelled industry at the first stage, where there is no technology gap between the firms at the time of investment. Let us denote hereafter the *first-stage* leader with the index $j = 1, 2$. We will also denote the first-stage laggard with the index $-j = 1, 2$, $-j \neq j$.

The cost-adjusted qualities at the second stage of the first-stage leader and laggard are equal to $n+\sigma+a_j x_j$ and $n-\sigma+a_{-j}x_{-j}$, respectively. Therefore, cost-adjusted qualities can change by a very small amount (i.e. "incrementally").[3] Crucially, in this model, depending on investments and shocks, the laggard of the first stage may become the leader in the second stage; this will be the case when $n_1 = n + \sigma + a_j x_j < n_2 = n - \sigma + a_{-j} x_{-j}$.

[3]Note that when the second-stage cost-adjusted qualities of the first-stage laggard and leader are given by $n+\sigma+2\sigma\mathbf{1}\{a_j x_j > \omega\}$ and $n-\sigma+2\sigma\mathbf{1}\{a_{-j}x_{-j} > \omega\}$ with $\sigma = 1, ..., S$ and ω is a constant, respectively, where $x \in [0, \infty)$ and G has a well-defined density g, we have instead a "technology-ladder" model. In that model, cost-adjusted qualities change in a discrete manner, firms can go up in the ladder one step at a time (hence, there is no "leapfrogging"), and every step of the ladder is of length 2σ. In such a model, the "probability of success" (or "research capacity") for firm i is $p_i \equiv 1 - G(\frac{\omega}{a_i})$. The probability of success is concave in a_i when $g(x) + \frac{d[xg(x)]}{dx} > 0$.

We know, then, that the equilibrium economic rents attained by duopolist $i = 1, 2$, equal

$$\Pi_i^* = \gamma \left(q_i^*\right)^2,$$

where

$$q_i^* = \frac{2\gamma n_i - \theta n_{-i}}{4\gamma^2 - \theta^2},$$

if $\theta \leq 2\gamma \frac{n_2}{n_1}$, while

$$q_1^* = \frac{n_1}{2\gamma}, q_2^* = 0,$$

if $2\gamma \frac{n_2}{n_1} < \theta < \gamma$.

Observe that the returns from investment depend on the realized second-stage cost-adjusted qualities of both firms n_1 and n_2 as well as the degree of substitutability θ.

Denote with $\mathbf{1}\{\Omega\}$ the index function that takes value one if Ω holds and value zero otherwise. We also impose the following assumptions that guarantee that the second-stage laggard's output is positive; that is,

Assumption A $\frac{2\gamma}{\theta} > \frac{n-\sigma}{n+\sigma} > \frac{\theta}{2\gamma}$ and $\overline{x} \leq \left[\frac{n[\frac{2\gamma}{\theta}-1]-\sigma[1+\frac{2\gamma}{\theta}]}{\overline{a}}\right]$.

To ensure that the problems of the first-stage laggard and leader are well-behaved, we also formulate

Assumption B $\;min\ \widehat{c}''(a) > K\frac{(2\gamma)^3\overline{x}^2}{(4\gamma^2-\theta^2)^2}$.

Before we move to the characterization of the equilibrium in an unlevelled industry we prove the following result, which will subsequently be useful.

Lemma $\left[K\frac{(2\gamma)^3 x_1^2}{(4\gamma^2-\theta^2)^2} - \widehat{c}''(a_1)\right]\left[K\frac{(2\gamma)^3 x_2^2}{(4\gamma^2-\theta^2)^2} - \widehat{c}''(a_2)\right] > \left[K\frac{(2\gamma)^2 x_1\theta x_2}{(4\gamma^2-\theta^2)^2}\right]^2$

Proof. Note that by Assumption B, $\overline{x} \geq x_1$, $\overline{x} \geq x_2$ and $\gamma > \theta$, we have:

$$[min\widehat{c}''(a)]^2 > \left[K\frac{(2\gamma)^3\,\overline{x}^2}{(4\gamma^2-\theta^2)^2}\right]^2 \Rightarrow$$

$$[-\widehat{c}''(a_1)]\,[-\widehat{c}''(a_2)] > \left[K\frac{(2\gamma)^2\,2\theta x_1 x_2}{(4\gamma^2-\theta^2)^2}\right]^2 \Rightarrow$$

$$\left[K\frac{(2\gamma)^3\,x_1^2}{(4\gamma^2-\theta^2)^2} - \widehat{c}''(a_1)\right]\left[K\frac{(2\gamma)^3\,x_2^2}{(4\gamma^2-\theta^2)^2} - \widehat{c}''(a_2)\right] > \left[K\frac{(2\gamma)^2\,\theta x_1 x_2}{(4\gamma^2-\theta^2)^2}\right]^2$$

■

D.1 The Investment Problem in an Unlevelled Industry

D.1.1 The First-Stage Laggard's Problem

Consider firm $j = 2$ taken to be the laggard in the first stage.

Suppose that $x_2 \leq \frac{2\sigma + a_1 x_1}{a_2}$ in which case we have that the first-stage laggard remains the laggard or catches up with the leader in the second stage (i.e. $n_2 = n - \sigma + a_2 x_2$). Let $X_2(a_2, a_1, x_1) \equiv \frac{\theta}{2\gamma}\frac{[n+\sigma+a_1x_1]}{a_2} - \frac{[n-\sigma]}{a_2}$. Assumption A implies that $\overline{x} \leq \left[\frac{n[\frac{2\gamma}{\theta}-1]-\sigma[1+\frac{2\gamma}{\theta}]}{\overline{a}}\right] = \frac{2\gamma}{\theta}\left[\frac{n[1-\frac{\theta}{2\gamma}]-\sigma[1+\frac{\theta}{2\gamma}]}{\overline{a}}\right]$ and hence that $x_1 \leq \frac{2\gamma}{\theta}\left[\frac{n[1-\frac{\theta}{2\gamma}]-\sigma[1+\frac{\theta}{2\gamma}]}{a_1}\right]$ and, thereby, $X_2(a_2, a_1, x_1) \leq 0 \leq x_2$ for any $x_2 \in [0, \overline{x})$. Recall from the oligopoly problem above that if $x_2 \geq X_2(a_2, a_1, x_1)$, then, $\theta \leq 2\gamma\frac{n_2}{n_1}$ and, thereby, the second-stage rents of the first-stage laggard are equal to $\gamma\left(\frac{2\gamma n_2 - \theta n_1}{4\gamma^2-\theta^2}\right)^2 = \pi_2(a_2x_2, a_1x_1) \equiv \gamma\left(\frac{n(2\gamma-\theta)-\sigma(2\gamma+\theta)+2\gamma a_2x_2-\theta a_1x_1}{4\gamma^2-\theta^2}\right)^2$.

Suppose instead that $x_2 > \frac{2\sigma+a_1x_1}{a_2}$ in which case we have that the first-stage laggard becomes the leader in the second stage (i.e. $n_1 = n - \sigma + a_2x_2$). Let $Y_2(a_2, a_1, x_1) \equiv \frac{2\gamma}{\theta}\frac{[n+\sigma+a_1x_1]}{a_2} - \frac{[n-\sigma]}{a_2}$. Assumption A implies that $\overline{x} \leq \left[\frac{n[\frac{2\gamma}{\theta}-1]-\sigma[1+\frac{2\gamma}{\theta}]}{\overline{a}}\right] \leq \frac{n[\frac{2\gamma}{\theta}-1]+\sigma[1+\frac{2\gamma}{\theta}]}{\overline{a}}$ and hence that $\overline{x} \leq \frac{n[\frac{2\gamma}{\theta}-1]+\sigma[1+\frac{2\gamma}{\theta}]+\frac{2\gamma}{\theta}a_1x_1}{a_1}$ and, thereby, $Y_2(a_2, a_1, x_1) \geq \overline{x} > x_2$ for any $x_2 \in [0, \overline{x})$. Recall from the oligopoly problem above that if $x_2 \leq Y_2(a_2, a_1, x_1) \equiv \frac{2\gamma}{\theta}\frac{[n+\sigma+a_1x_1]}{a_2} - \frac{[n-\sigma]}{a_2}$, then, $\theta \leq 2\gamma\frac{n_2}{n_1}$ and, thereby, the rents in the second stage of the first-stage laggard are equal to $\gamma\left(\frac{2\gamma n_1-\theta n_2}{4\gamma^2-\theta^2}\right)^2 = \pi_2(a_2x_2, a_1x_1)$.

It follows that the laggard of the first stage maximizes with respect to $a_2 \geq 0$ the following expected profits:

$$\int_0^{\overline{x}} \mathbf{1}\{\frac{2\sigma + a_1x_1}{a_2} < \overline{x}\} \int_{\frac{2\sigma+a_1x_1}{a_2}}^{\overline{x}} \pi_2(a_2x_2, a_1x_1)dG(x_2)dG(x_1)+$$

$$\int_0^{\overline{x}} \int_0^{min\{\overline{x}, \frac{2\sigma+a_1x_1}{a_2}\}} \pi_2(a_2x_2, a_1x_1)dG(x_2)dG(x_1) - \frac{\widehat{c}(a_2)}{K} =$$

$$\int_0^{\overline{x}} \mathbf{1}\{\frac{2\sigma + a_1x_1}{a_2} < \overline{x}\} \int_0^{\overline{x}} \pi_2(a_2x_2, a_1x_1)dG(x_2)dG(x_1)+$$

$$\int_0^{\overline{x}} \mathbf{1}\{\frac{2\sigma + a_1x_1}{a_2} \geq \overline{x}\} \int_0^{\overline{x}} \pi_2(a_2x_2, a_1x_1)dG(x_2)dG(x_1) - \frac{\widehat{c}(a_2)}{K} =$$

$$\int_0^{\overline{x}} \int_0^{\overline{x}} \pi_2(a_2x_2, a_1x_1)dG(x_2)dG(x_1) - \frac{\widehat{c}(a_2)}{K}.$$

Assumption B ensures that

$$\frac{\partial^2}{\partial a_2^2}\left[K\pi_2(a_2x_2, a_1x_1) - \widehat{c}(a_2)\right] = K\left(2\gamma\right)^2 x_2 \left(\frac{2\gamma x_2}{\left(4\gamma^2-\theta^2\right)^2}\right) - \widehat{c}''(a_2) < 0,$$

and so the above objective function is concave (as in our "ladder-type" model).[4] The first-order condition (at an interior solution) is thus

$$K\int_0^{\overline{x}} \int_0^{\overline{x}} \frac{\partial}{\partial a_2}\pi_2(a_2x_2, a_1x_1)dG(x_2)dG(x_1) = \widehat{c}'(a_2).$$

Note now that

$$\frac{\partial}{\partial a_2}\pi_2(a_2x_2, a_1x_1) = 2\gamma\left(\frac{n(2\gamma-\theta) - \sigma(2\gamma+\theta) + 2\gamma a_2x_2 - \theta a_1x_1}{4\gamma^2-\theta^2}\right)\frac{2\gamma x_2}{4\gamma^2-\theta^2} > 0,$$

and hence we have that

$$\frac{\partial^2}{\partial a_2\partial a_1}K\pi_2(a_2x_2, a_1x_1) = K\left(2\gamma\right)^2 x_2\left(\frac{-\theta x_1}{\left(4\gamma^2-\theta^2\right)^2}\right) < 0$$

as in our "ladder-type" model and that

$$\frac{\partial^2}{\partial a_2\partial\theta}K\pi_2(a_2x_2, a_1x_1) = K\left(2\gamma\right)^2 x_2\left(\frac{-(n+\sigma+a_1x_1)}{\left(4\gamma^2-\theta^2\right)^2} + 4\theta\frac{n(2\gamma-\theta) - \sigma(2\gamma+\theta) + 2\gamma a_2x_2 - \theta a_1x_1}{\left(4\gamma^2-\theta^2\right)^3}\right).$$

D.1.2 The First-Stage Leader's Problem

Consider firm $j = 1$ taken to be the leader in the first stage.

Suppose that $x_1 \geq \frac{-2\sigma+a_2x_2}{a_1}$ in which case we have that the first-stage leader stays the leader or is caught up by the laggard in the second stage (i.e. $n_1 = n + \sigma + a_1x_1$). Let $Y_1(a_1, a_2, x_2) \equiv \frac{2\gamma}{\theta}\frac{[n-\sigma+a_2x_2]}{a_1} - \frac{[n+\sigma]}{a_1}$. Assumption A implies that $\overline{x} \leq \left[\frac{n[\frac{2\gamma}{\theta}-1]-\sigma[1+\frac{2\gamma}{\theta}]}{\overline{a}}\right]$ and hence that $\overline{x} \leq \frac{n[\frac{2\gamma}{\theta}-1]-\sigma[1+\frac{2\gamma}{\theta}]+\frac{2\gamma}{\theta}a_2x_2}{a_1}$ and, thereby, $Y_1(a_1, a_2, x_2) \geq \overline{x} > x_1$ for any $x_1 \in [0, \overline{x})$. Recall from the oligopoly problem above that if $x_1 \leq Y_1(a_1, a_2, x_2)$, then,

[4] Notice that under certainty, the above objective function becomes $\pi_2(a_2, a_1) - \frac{\widehat{c}(a_2)}{K}$.

$\theta \leq 2\gamma\frac{n_2}{n_1}$ and, thereby, the rents in the second stage of the first-stage leader are equal to $\gamma\left(\frac{2\gamma n_1-\theta n_2}{4\gamma^2-\theta^2}\right)^2 = \pi_1(a_1x_1, a_2x_2) \equiv \gamma\left(\frac{n(2\gamma-\theta)+\sigma(2\gamma+\theta)+2\gamma a_1x_1-\theta a_2x_2}{4\gamma^2-\theta^2}\right)^2$.

Suppose that $x_1 < \frac{-2\sigma+a_2x_2}{a_1}$ in which case we have that the first-stage leader becomes the laggard in the second stage (i.e. $n_2 = n+\sigma+a_1x_1$). Let $X_1(a_1,a_2,x_2) \equiv \frac{\theta}{2\gamma}\frac{[n-\sigma+a_2x_2]}{a_1} - \frac{[n+\sigma]}{a_1}$. Assumption A implies that $\overline{x} \leq \left[\frac{n[\frac{2\gamma}{\theta}-1]-\sigma[1+\frac{2\gamma}{\theta}]}{\overline{a}}\right] = \left[\frac{n[\frac{2\gamma}{\theta}-1]+\sigma[1+\frac{2\gamma}{\theta}]}{\overline{a}}\right]$ and hence that $x_2 \leq \left[\frac{n[\frac{2\gamma}{\theta}-1]+\sigma[1+\frac{2\gamma}{\theta}]}{a_2}\right]$ and, thereby, $X_1(a_1,a_2,x_2) \leq 0 \leq x_1$ for any $x_1 \in [0,\overline{x})$. Recall from the oligopoly problem above that if $x_1 \geq X_1(a_1,a_2,x_2)$, then, $\theta \leq 2\gamma\frac{n_2}{n_1}$ and, thereby, the second-stage rents of the first-stage leader are equal to $\gamma\left(\frac{2\gamma n_2-\theta n_1}{4\gamma^2-\theta^2}\right)^2 = \pi_1(a_1x_1, a_2x_2)$.

It follows that the leader of the first stage maximizes with respect to $a_1 \geq 0$ the following expected profits

$$\int_0^{\overline{x}} \mathbf{1}\{\frac{-2\sigma+a_2x_2}{a_1} < \overline{x}\} \int_{max\{0,\frac{-2\sigma+a_2x_2}{a_1}\}}^{\overline{x}} \pi_1(a_1x_1,a_2x_2)dG(x_1)dG(x_2) +$$

$$\int_0^{\overline{x}} \mathbf{1}\{\frac{-2\sigma+a_2x_2}{a_1} > 0\} \int_0^{min\{\overline{x},\frac{-2\sigma+a_2x_2}{a_1}\}} \pi_1(a_1x_1,a_2x_2)dG(x_1)dG(x_2) - \frac{\widehat{c}(a_2)}{K} =$$

$$\int_0^{\overline{x}} \mathbf{1}\{0 < \frac{-2\sigma+a_1x_1}{a_2} < \overline{x}\} \int_0^{\overline{x}} \pi_1(a_1x_1,a_2x_2)dG(x_1)dG(x_2) +$$

$$\int_0^{\overline{x}} \mathbf{1}\{\frac{-2\sigma+a_1x_1}{a_2} \leq 0 \text{ or } \frac{-2\sigma+a_1x_1}{a_2} \geq \overline{x}\} \int_0^{\overline{x}} \pi_1(a_1x_1,a_2x_2)dG(x_1)dG(x_2) - \frac{\widehat{c}(a_1)}{K} =$$

$$\int_0^{\overline{x}} \int_0^{\overline{x}} \pi_1(a_1x_1,a_2x_2)dG(x_1)dG(x_2) - \frac{\widehat{c}(a_1)}{K}.$$

Assumption B ensures that

$$\frac{\partial^2}{\partial a_1^2}\left[K\pi_1(a_1x_1,a_2x_2) - \widehat{c}(a_1)\right] = K\left(2\gamma\right)^2 x_1 \left(\frac{2\gamma x_1}{\left(4\gamma^2-\theta^2\right)^2}\right) - \widehat{c}''(a_1) < 0,$$

and so the above objective function is concave (as in our "ladder-type" model).[5] The first-

[5]Notice that under certainty, the above objective function becomes $\pi_1(a_1,a_2) - \frac{\widehat{c}(a_1)}{K}$.

order condition (at an interior solution) is thus

$$K\int_0^{\overline{x}}\int_0^{\overline{x}}\frac{\partial}{\partial a_1}\pi_1(a_1x_1,a_2x_2)dG(x_1)dG(x_2)=\widehat{c}'(a_1).$$

Note now that

$$\frac{\partial}{\partial a_1}\pi_1(a_1x_1,a_2x_2)=2\gamma\left(\frac{n(2\gamma-\theta)+\sigma(2\gamma+\theta)+2\gamma a_1x_1-\theta a_2x_2}{4\gamma^2-\theta^2}\right)\frac{2\gamma x_1}{4\gamma^2-\theta^2}>0$$

and hence we have that

$$\frac{\partial^2}{\partial a_1\partial a_2}K\pi_1(a_1x_1,a_2x_2)=K\left(2\gamma\right)^2x_1\left(\frac{-\theta x_2}{\left(4\gamma^2-\theta^2\right)^2}\right)<0,$$

in contrast to our "ladder-type" model and that

$$\frac{\partial^2}{\partial a_1\partial\theta}\pi_1(a_1x_1,a_2x_2)=\left(2\gamma\right)^2x_1\left(\frac{-(n-\sigma+a_2x_2)}{\left(4\gamma^2-\theta^2\right)^2}+4\theta\frac{n(2\gamma-\theta)+\sigma(2\gamma+\theta)+2\gamma a_1x_1-\theta a_2x_2}{\left(4\gamma^2-\theta^2\right)^3}\right).$$

D.1.3 Equilibrium Investment

The Lemma implies that the Implicit Function Theorem is satisfied, and so we can find the impact of a higher degree of product substitutability on the first-stage laggard's investment by using the Cramer's rule. In more detail we have that

$$\left[\begin{array}{cc}\int_0^{\overline{x}}\int_0^{\overline{x}}\frac{\partial^2}{\partial a_1^2}\left[K\pi_1(a_1x_1,a_2x_2)-\widehat{c}(a_1)\right]dG(x_1)dG(x_2) & \int_0^{\overline{x}}\int_0^{\overline{x}}\frac{\partial^2}{\partial a_1\partial a_2}K\pi_1(a_1x_1,a_2x_2)dG(x_1)dG(x_2)\\ \int_0^{\overline{x}}\int_0^{\overline{x}}\frac{\partial^2}{\partial a_2\partial a_1}K\pi_2(a_2x_2,a_1x_1)dG(x_1)dG(x_2) & \int_0^{\overline{x}}\int_0^{\overline{x}}\frac{\partial^2}{\partial a_2^2}\left[K\pi_2(a_2x_2,a_1x_1)-\widehat{c}(a_2)\right]dG(x_1)dG(x_2)\end{array}\right]\times$$

$$\left[\begin{array}{c}\frac{\partial a_1}{\partial\theta}\\ \frac{\partial a_2}{\partial\theta}\end{array}\right]=\left[\begin{array}{c}-\int_0^{\overline{x}}\int_0^{\overline{x}}\frac{\partial^2}{\partial a_1\partial\theta}K\pi_1(a_1x_1,a_2x_2)dG(x_1)dG(x_2)\\ -\int_0^{\overline{x}}\int_0^{\overline{x}}\frac{\partial^2}{\partial a_2\partial\theta}K\pi_2(a_2x_2,a_1x_1)dG(x_1)dG(x_2)\end{array}\right].$$

Note first that the determinant of the Jacobian of the system of equations defined by the leader's and laggard's first-order conditions above is positive due to the Lemma. Consequently, to have that higher degree of product substitutability leads to lower investment for the first-stage laggard given opponent's investment (as it is the case in our model) we need that

$$\frac{\partial^2}{\partial a_1^2}\left[K\pi_1(a_1x_1,a_2x_2)-\widehat{c}(a_1)\right]\frac{\partial^2}{\partial a_2\partial\theta}K\pi_2(a_2x_2,a_1x_1)-\frac{\partial^2}{\partial a_2\partial a_1}K\pi_2(a_2x_2,a_1x_1)K\frac{\partial^2}{\partial a_1\partial\theta}\pi_1(a_1x_1,a_2x_2)>0.$$

Recall from our analysis above of the forms' first-order conditions that $\frac{\partial^2}{\partial a_2 \partial a_1}\pi_2(a_2x_2, a_1x_1) < 0$, and that, by Assumption B, we have that $\frac{\partial^2}{\partial a_1^2}\left[K\pi_1(a_1x_1, a_2x_2) - \widehat{c}(a_1)\right] < 0$. Therefore, a sufficient condition for the above inequality is that $\frac{\partial^2}{\partial a_2 \partial \theta}\pi_2(a_2x_2, a_1x_1) < 0$ and $\frac{\partial^2}{\partial a_1 \partial \theta}\pi_1(a_1x_1, a_2x_2) > 0$. These are the counterparts of assumptions (2) and (3) in the main text of our paper.

In what follows, we derive conditions on the parameters of the model that ensure these two sufficient conditions.

We start by observing that $\frac{\partial^2}{\partial a_2 \partial \theta}\pi_2(a_2x_2, a_1x_1) < 0$ can be re-written as

$$4\theta\left(\frac{n(2\gamma-\theta)-\sigma(2\gamma+\theta)+2\gamma a_2x_2-\theta a_1x_1}{4\gamma^2-\theta^2}\right) < n+\sigma+a_1x_1.$$

A **sufficient condition** for this is that (a) $\overline{ax}$ is high enough so that $4\left(\frac{n-\sigma 3+2\overline{ax}}{3}\right) > n+\sigma$, and (b) γ is sufficiently high so that

$$4\theta\left(\frac{n(2\gamma-\theta)-\sigma(2\gamma+\theta)+2\gamma\overline{ax}}{4\gamma^2-\theta^2}\right) < n+\sigma.$$

To see this, recall first that $\gamma > \theta$, and observe that the limit as γ becomes very large of the left-hand side of the above inequality is zero (hence lower than $n+\sigma$), and that $4\left(\frac{n-\sigma 3+2\overline{ax}}{3}\right) > n+\sigma$ implies that the left-hand side of the above inequality when evaluated at $\gamma = \theta$ is higher than $n+\sigma$. Consequently, the Intermediate Value Theorem implies that there is a threshold value of γ such that the above inequality is satisfied for all higher values of γ. Second, observe that the above inequality implies directly that

$$4\theta\left(\frac{n(2\gamma-\theta)-\sigma(2\gamma+\theta)+2\gamma a_2x_2-\theta a_1x_1}{4\gamma^2-\theta^2}\right) < n+\sigma+a_1x_1$$

as desired.

Next, observe that $\frac{\partial^2}{\partial a_1 \partial \theta}\pi_1(a_1x_1, a_2x_2) > 0$ can be re-written as

$$4\theta\left(\frac{n(2\gamma-\theta)+\sigma(2\gamma+\theta)+2\gamma a_1x_1-\theta a_2x_2}{4\gamma^2-\theta^2}\right) > n-\sigma+a_2x_2.$$

A **sufficient condition** for this is that (a) $4\theta\left(\frac{4S\gamma-\theta\overline{ax}}{4\gamma^2-\theta^2}\right) > \overline{ax}$, (b) γ is sufficiently high so that $\frac{4\theta}{2\gamma+\theta} < 1$, and (c) n is sufficiently low so that

$$4\theta\left(\frac{n(2\gamma-\theta)+\sigma(2\gamma+\theta)-\theta\overline{ax}}{4\gamma^2-\theta^2}\right) > n-\sigma+\overline{ax}.$$

To see this, note first that both sides of the above inequality are linear in n. The corresponding slope of the left-hand side is equal to $\frac{4\theta(2\gamma-\theta)}{4\gamma^2-\theta^2} = \frac{4\theta}{2\gamma+\theta}$, which goes to zero as γ becomes very large, while the corresponding slope of the right-hand side of the above inequality is equal to one. Therefore, if γ is sufficiently high (i.e. specifically, $\gamma > \frac{3}{2}\theta$), we have that as n becomes very large the left-hand side of the above inequality ends up being lower than the right-hand side of the above inequality. Recall next that $n > \sigma$ and, then, observe that the limit as n converges to σ of the left-hand side of the above inequality is equal to $4\theta\left(\frac{4\sigma\gamma-\theta\overline{ax}}{4\gamma^2-\theta^2}\right)$, while the corresponding limit of the right-hand side of the above inequality is equal to $\overline{ax}$. Observe now that $4\theta\left(\frac{4S\gamma-\theta\overline{ax}}{4\gamma^2-\theta^2}\right) > \overline{ax}$ implies that there is a threshold value of σ such that for sufficiently high σ, we have that $4\theta\left(\frac{4\sigma\gamma-\theta\overline{ax}}{4\gamma^2-\theta^2}\right) > \overline{ax}$. Consequently,[6] the Intermediate Value Theorem implies that there is a threshold value of n such that the above inequality is satisfied for all lower values of n. Finally, observe that the above inequality implies directly that

$$4\theta\left(\frac{n(2\gamma-\theta)+\sigma(2\gamma+\theta)+2\gamma a_1x_1-\theta a_2x_2}{4\gamma^2-\theta^2}\right) > n-\sigma+a_2x_2$$

as desired.

To summarize, for sufficiently high $\overline{ax}$, σ, γ and sufficiently low $n(>\sigma)$, our result under the "ladder-type" model that a higher degree of product substitutability reduces the R&D investment of the first-stage laggard carries forward to the current model of incremental investment as well (regardless of whether the investment outcome is certain or stochastic).[7]

D.2 The Investment Problem in a Levelled Industry

This problem is formulated by setting $\sigma = 0$ in the above two problems, where $X_1(z,w,x) = X_2(z,w,x) = X(z,w,x) \equiv \frac{\theta}{2\gamma}\frac{[n+wx]}{z} - \frac{n}{z}$ and $Y_1(z,w,x) = Y_2(z,w,x) = Y(z,w,x) \equiv \frac{2\gamma}{\theta}\frac{[n+wx]}{z} - \frac{n}{z}$ and $\pi_1(zx,wy) = \pi_2(zx,wy) = \pi(zx,wy) \equiv \gamma\left(\frac{n(2\gamma-\theta)+2\gamma zx-\theta wy}{4\gamma^2-\theta^2}\right)^2$; that is, each firm $j=1,2$, while taking as given the opponent's choice a_{-j}, maximizes with respect

[6]Notice here that, for $\theta \to \gamma^+$, $4\theta\left(\frac{4\sigma\gamma-\theta\overline{ax}}{4\gamma^2-\theta^2}\right) > \overline{ax}$ becomes $4\left(\frac{4\sigma-\overline{ax}}{3}\right) > \overline{ax}$ and hence $\sigma > \frac{7}{16}\overline{ax}$. This is compatible, under $n \to \sigma^-$, with the earlier requirement that $4\left(\frac{n-\sigma3+2\overline{ax}}{3}\right) > n+\sigma$. To see this, note that when $n=\sigma$, the latter inequality becomes $\sigma < \frac{8}{14}\overline{ax}$, where $\frac{8}{14} > \frac{7}{16}$.

[7]From Cramer's rule and the above sufficient conditions that ensure $\frac{\partial^2}{\partial a_2\partial\theta}\pi_2(a_2x_2,a_1x_1) < 0$ and $\frac{\partial^2}{\partial a_1\partial\theta}\pi_1(a_1x_1,a_2x_2) > 0$, one can also easily see that, in contrast to our model, the effect of a higher degree of product substitutability on the investment of the first-stage leader is positive. This follows from the fact that here, as we have shown above, we have that $\frac{\partial^2}{\partial a_1\partial a_2}\pi_1(a_1x_1,a_2x_2) < 0$.

to $a_j \geq 0$ the following expected profits

$$\int_0^{\overline{x}} \int_0^{\overline{x}} \pi_j(a_j x_j, a_{-j}x_{-j}) dG(x_j) dG(x_{-j}) - \frac{\widehat{c}(a_j)}{K}$$

with $a_{-j} = a_j$ in a symmetric equilibrium.

Assumption B ensures that

$$\frac{\partial^2}{\partial a_j^2} \left[K\pi_j(a_j x_j, a_{-j}x_{-j}) - \widehat{c}(a_j) \right] = K\left(2\gamma\right)^2 x_j \left(\frac{2\gamma x_j}{\left(4\gamma^2 - \theta^2\right)^2} \right) - \widehat{c}''(a_j) < 0,$$

and so the above objective function is concave (as in our "ladder-type" model).[8] The first-order condition (at an interior solution) is thus

$$K \int_0^{\overline{x}} \int_0^{\overline{x}} \frac{\partial}{\partial a_j} \pi_j(a_j x_j, a_{-j}x_{-j}) dG(x_j) dG(x_{-j}) = \widehat{c}'(a_j).$$

Note now that

$$\frac{\partial}{\partial a_j}\pi_j(a_j x_j, a_{-j}x_{-j}) = 2\gamma \left(\frac{n(2\gamma - \theta) + 2\gamma a_j x_j - \theta a_{-j}x_{-j}}{4\gamma^2 - \theta^2} \right) \frac{2\gamma x_j}{4\gamma^2 - \theta^2} > 0.$$

Setting $a = a_j = a_{-j}$, the above first-order condition becomes

$$\int_0^{\overline{x}} \int_0^{\overline{x}} K\left(2\gamma\right)^2 x_j \left(\frac{n(2\gamma - \theta) + 2\gamma a x_j - \theta a x_{-j}}{\left(4\gamma^2 - \theta^2\right)^2} \right) dG(x_j) dG(x_{-j}) = \widehat{c}'(a) \tag{9}$$

and thus, using the Implicit Function Theorem, we have that

$$\frac{\partial a}{\partial \theta} = -\frac{\int_0^{\overline{x}} \int_0^{\overline{x}} K\left(2\gamma\right)^2 x_j \left(\frac{-(n + a x_{-j})}{\left(4\gamma^2 - \theta^2\right)^2} + 4\theta \frac{n(2\gamma-\theta) + 2\gamma a x_j - \theta a x_{-j}}{\left(4\gamma^2 - \theta^2\right)^3} \right) dG(x_j) dG(x_{-j})}{\int_0^{\overline{x}} K\left(2\gamma\right)^2 x_j \left(\frac{2\gamma x_j - \theta x_{-j}}{\left(4\gamma^2 - \theta^2\right)^2} \right) dG(x_j) - \widehat{c}''(a)}.$$

The denominator is negative by Assumption B, whereas the numerator cannot be signed without further restrictions on the primitives.[9] On one hand, if $4\theta \frac{n(2\gamma-\theta) + 2\gamma \overline{a}\overline{x}}{(4\gamma^2 - \theta^2)} \leq n$, then, $4\theta \frac{n(2\gamma-\theta) + 2\gamma a x_j - \theta a x_{-j}}{(4\gamma^2 - \theta^2)} < n + a x_{-j}$ for any profile of shocks, and so the numerator above and, thereby, $\frac{\partial a}{\partial \theta}$ is always negative in contrast to our "ladder-type" model. If, on the other hand, $4\theta \frac{n(2\gamma-\theta) + 2\gamma \overline{a}\overline{x}}{(4\gamma^2 - \theta^2)} > n$, then, the sign of the numerator depends on the distribution G and the

[8] Notice that under certainty, the above objective function becomes $\pi_j(a_j, a_{-j}) - \frac{\widehat{c}(a_j)}{K}$.

[9] Our conditions/assumptions so far do not guarantee the sign of the numerator above.

size of a. In turn, the level of a and, thereby, the sign of the numerator above is affected by the level of K. Specifically, we have from (9) that (as in our "ladder-type" model) the higher the K, the higher a is in the symmetric equilibrium, and vice versa. Therefore, as in our "ladder-type" model, the effect of an increase in product substitutability on R&D investment in a levelled industry can be non-monotone and, crucially, dependent on the level of R&D productivity K. Determining, however, the exact relationship is more complicated in this model than the one in our main text.

E Technical Details Omitted from Section 2

E.1 The First-Stage Unlevelled Industry

The investment problem of the laggard (firm = 2) is to maximize with respect to p_2:

$$(1-p_2)\pi_l + p_2\left[p_1^*\pi_l + (1-p_1^*)\pi_s\right] - C(p_2, K) =$$

$$\pi_l + p_2(1-p_1^*)(\pi_s - \pi_l) - \frac{c(p_2)}{K}.$$

Based on the model's assumptions, this problem is well-defined. To understand this problem note that increasing marginally the research capacity of firm $i = 2$ leads to a higher cost by $c'(p_2)/K$ units, and to an increase in expected rents by $(1-p_1^*)(\pi_s - \pi_l)$ units. The latter increase is the gain from being in a levelled industry in the second stage, which occurs when firm $i = 2$ innovates and the rival does not succeed in innovating.

Taking the first-order condition with respect to p_2, we have at an interior solution (i.e. when $p_2^* > 0$) that

$$K(1-p_1^*)(\pi_s - \pi_l) = c'(p_2^*). \tag{10}$$

The optimal research capacity of the laggard p_2^* is increasing in its relative marginal benefit $K(1-p_1^*)(\pi_s - \pi_l)$. Clearly, if $K(1-p_1^*)(\pi_s - \pi_l) \leq c'(0)$ then $p_2^* = 0$.

The problem of the leader (firm = 1), in turn, is to maximize with respect to p_1:

$$(1-p_1)[p_2^*\pi_s + (1-p_2^*)\pi_h] + p_1\pi_h - C(p_1, K) =$$

$$\pi_h - (1-p_1)p_2^*(\pi_h - \pi_s) - \frac{c(p_1)}{K}.$$

This problem is well-defined as well. As with the laggard's problem, increasing marginally the research capacity of firm $i = 1$ leads to a higher cost by $c'(p_1)/K$ units, and to an increase in expected rents by $p_2^*(\pi_h - \pi_s)$ units. The latter increase is the gain from avoiding being in a levelled industry in the second stage, which will occur when firm $i = 1$ fails to innovate and the rival succeeds in innovating. Taking the first-order condition with respect to p_1, we have at an interior solution (i.e. when $p_1^* > 0$) that

$$Kp_2^*(\pi_h - \pi_s) = c'(p_1^*). \tag{11}$$

The optimal research capacity of the leader p_1^* is increasing in its relative marginal benefit $Kp_2^*(\pi_h - \pi_s)$. If $Kp_2^*(\pi_h - \pi_s) \leq c'(0)$, we, then, have that $p_1^* = 0$.

Observe thus that if

$$c'(0) \geq K(\pi_s - \pi_l), \tag{12}$$

then, at equilibrium, there is no R&D investment by either the laggard or the leader. In this case, the laggard's marginal cost from investment at any strictly positive level of investment is higher than the highest possible marginal benefit from investment, making zero investment optimal. This, in turn, implies that zero investment is optimal for the leader as well (i.e. $p_1^* = p_2^* = 0$).

We turn to the case when only the laggard chooses a strictly positive research capacity (which requires that the above inequality does not hold). Let us rewrite the first-order condition of the laggard (10) (after dropping the asterisks) as

$$p_1 = 1 - \frac{c'(p_2)}{K(\pi_s - \pi_l)}. \tag{13}$$

Viewing this as defining a function $p_1(p_2)$, we see that it is decreasing and concave in p_2. Moreover, it goes to minus infinity as p_2 approaches 1. In addition, when $p_2 = c'^{-1}(K(\pi_s - \pi_l))$, we have that $p_1 = 0$. Clearly, then, if

$$c'(0) < K(\pi_s - \pi_l), \tag{14}$$

the highest possible research capacity for the laggard, which is consistent with equilibrium behavior and $p_1 \geq 0$, is equal to $c'^{-1}K(\pi_s - \pi_l))$. As a direct consequence from the leader's incentives as described by (11), if also

$$c'^{-1}K(\pi_s - \pi_l)) \leq \frac{c'(0)}{K(\pi_h - \pi_s)}, \tag{15}$$

then, the leader's marginal cost from investment at any strictly positive level of investment is higher than the highest possible marginal benefit from investment, making zero investment optimal. Thus, in the case where the above two inequalities are satisfied, we have that $p_1^* = 0$ and from (13): $p_2^* = c'^{-1}K(\pi_s - \pi_l)) > 0$.

Turning to the case where both firms invest (which requires that from the above two inequalities the last one does not hold), we rewrite the first-order condition of the leader (11) (after dropping the asterisks) as

$$p_2 = \frac{c'(p_1)}{K(\pi_h - \pi_s)}. \tag{16}$$

Viewing this as defining a function $p_2(p_1)$, we observe that it is increasing and convex in p_1. Moreover, it goes to infinity as p_1 approaches 1. A diagrammatic inspection of (13) and (16) is enough to convince the reader that if

$$c'(0) < K(\pi_s - \pi_l)$$

(and thereby that $c'^{-1}(K(\pi_s - \pi_l)) > 0$), and

$$c'^{-1}(K(\pi_s - \pi_l)) > \frac{c'(0)}{K(\pi_h - \pi_s)}, \tag{17}$$

then, these two curves have a unique intersection at strictly positive values of p_1^* and p_2^* (together defining the equilibrium profile of research capacities).

This completes the characterization of equilibria when the industry is unlevelled in the first stage. We have identified three cases: (a) where neither the leader nor the laggard invests, (b) where the laggard but not the leader invests, and (c) where both the leader and the laggard invest.

We restrict attention to the (more interesting) case where there is strictly positive investment from the laggard in the equilibrium. To start with, we observe that if $p_1^* = 0$, then, the laggard's R&D investment decreases with product substitutability: in this case $p_2^* = c'^{-1}(K(\pi_s - \pi_l))$, where, by assumption, $c'^{-1}(K(\pi_s - \pi_l))$ is decreasing in θ.

Turning to the case where both firms invest, to find the effect of θ on the equilibrium research capacities, we need to use the Implicit Function Theorem. So, dropping the asterisks for notational simplicity, and using (10) and (11), we have that

$$\begin{bmatrix} -c''(p_1) & K(\pi_h - \pi_s) \\ -K(\pi_s - \pi_l) & -c''(p_2) \end{bmatrix} \begin{bmatrix} \partial p_1/\partial\theta \\ \partial p_2/\partial\theta \end{bmatrix} = \begin{bmatrix} -Kp_2\frac{\partial[\pi_h - \pi_s]}{\partial\theta} \\ -K(1-p_1)\frac{\partial[\pi_s - \pi_l]}{\partial\theta} \end{bmatrix}.$$

Therefore, the effect of θ on laggard's R&D investment is determined by the sign of

$$\partial p_2/\partial\theta =$$

$$\frac{c''(p_1)K(1-p_1)\frac{\partial[\pi_s - \pi_l]}{\partial\theta} - K(\pi_s - \pi_l)Kp_2\frac{\partial[\pi_h - \pi_s]}{\partial\theta}}{c''(p_1)c''(p_2) + K(\pi_h - \pi_s)K(\pi_s - \pi_l)}.$$

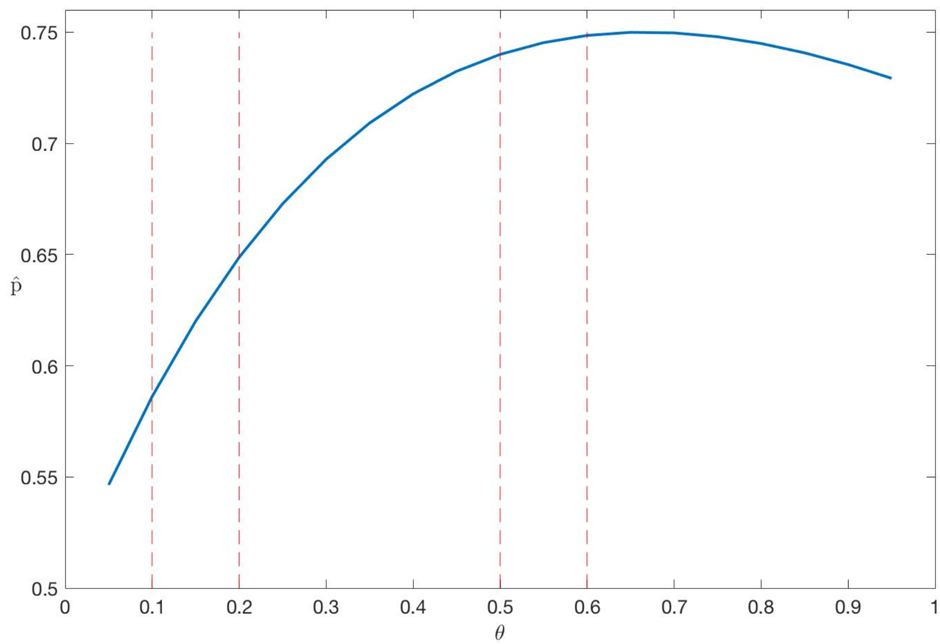
Note that the denominator is positive by the convexity of the cost function and assumption (1). In addition, the numerator is negative by the convexity of the cost function and assumptions (2) and (3). Therefore, the laggard's R&D investment decreases with product substitutability in this case as well.[10]

[10]One can easily see from the above that the effect of higher θ on the leader's investment (i.e. $\partial p_1/\partial\theta$) cannot be signed without further assumptions on the primitives of the model.

F Dependence of $\widehat{p}$ on θ

Recall that, at the levelled duopoly, we cannot say more about the monotonicity properties of p^* with respect to the degree of product substitutability θ, unless we impose more assumptions on the dependence of the industry's rents profile on θ (see Subsection 2.1.2). Based on the industry's rents profile chosen in the experiments, the dependence of $\widehat{p}$ on θ is shown in Figure 1. The relation is non-monotone. However, for the θs chosen in the experiments (i.e. $\theta \in \{0.1, 0.2, 0.5, 0.6\}$), $\widehat{p}$ is increasing in θ. In fact, up to $\theta = 0.66$, $\widehat{p}$ is increasing in θ.

Figure 1: Dependence of $\widehat{p}$ on θ



Notes: We provide the relation between $\widehat{p}$ and θ. The most intense level of product substitutability chosen in the experiments is $\theta = 0.6$. The vertical dotted lines indicate the four levels of product substitutability chosen in the experiments. Up to $\theta = 0.66$, $\widehat{p}$ is increasing in θ.

G Mann-Whitney-Wilcoxon Tests

To complement our regression analysis, we use the Mann-Whitney-Wilcoxon (non-parametric) tests to determine any difference in the distributions of investment choices across the selected levels of product substitutability. Specifically, the null hypothesis states that there is no difference in the distribution of investments so that it is equally likely that a randomly selected value from one level of θ is less than or greater than a randomly selected value from another level of θ. The results are displayed in Table 1. In Panel A, we report the p-values in the laggard scenario, in Panel B, we report the p-values in the levelled scenario and, in Panel C, we report the p-values in the leader scenario.

The regression results of Panel A in Table 3 in the manuscript show a significant negative effect of product substitutability on investment choices of laggards both when the level of R&D productivity is low and when it is high. Similar results are found using the Mann-Whitney-Wilcoxon tests. Specifically, the statistics reported in Panel A of Table 1 reject the null for the pairwise comparisons of $\theta = 0.1$ vs. $\theta = 0.5$ (p-value $= 0.044$) and $\theta = 0.1$ vs. $\theta = 0.6$ (p-value $= 0.061$) when the level of R&D productivity is low, whereas when the level of R&D productivity is high, the null is rejected for the levels $\theta = 0.1$ vs. $\theta = 0.6$ (p-value $= 0.008$) and $\theta = 0.2$ vs. $\theta = 0.6$ (p-value $= 0.033$). Furthermore, the regression results show that investment by levelled firms increases with product substitutability when the level of R&D productivity is low. These results are corroborated in the Mann-Whitney-Wilcoxon tests in Panel B of Table 1. Specifically, the tests reject the null that the distribution of investments is equal when the level of R&D productivity is low in the pairwise comparisons of $\theta = 0.1$ vs. $\theta = 0.6$ (p-value $= 0.002$) and $\theta = 0.5$ vs. $\theta = 0.6$ (p-value $= 0.051$). We also examine the U-shaped but practically 'flat' relation between investment and product substitutability identified by the proposed model when the level of R&D productivity is high. Similar to the regression results, when the level of R&D productivity is high, none of the p-values in the pairwise comparisons is statistically significant; that is, we cannot reject the null that the distribution of investments across paired product substitutability levels is equal. Finally, in Panel C of Table 1, we report the p-values of the Mann-Whitney-Wilcoxon tests to determine any difference in the distribution of investments of the leaders across the selected levels of product substitutability. In the pairwise comparisons, when R&D productivity is high, we confirm that none of the p-values are lower than the 10% significance level.

Table 1: Mann-Whitney-Wilcoxon Tests on Investment

Panel A: Laggard		
	Low R&D Productivity	High R&D Productivity
Alternative Hypothesis:	$Investment_i \neq Investment_j$	
	p-values	p-values
$\theta = 0.1$ vs. $\theta = 0.2$	0.392	0.904
$\theta = 0.1$ vs. $\theta = 0.5$	0.044	0.164
$\theta = 0.1$ vs. $\theta = 0.6$	0.061	0.008
$\theta = 0.2$ vs. $\theta = 0.5$	0.238	0.367
$\theta = 0.2$ vs. $\theta = 0.6$	0.157	0.033
$\theta = 0.5$ vs. $\theta = 0.6$	0.793	0.173
Panel B: Levelled		
	Low R&D Productivity	High R&D Productivity
Alternative Hypothesis:	$Investment_i \neq Investment_j$	
	p-values	p-values
$\theta = 0.1$ vs. $\theta = 0.2$	0.122	0.627
$\theta = 0.1$ vs. $\theta = 0.5$	0.105	0.238
$\theta = 0.1$ vs. $\theta = 0.6$	0.002	0.522
$\theta = 0.2$ vs. $\theta = 0.5$	0.856	0.576
$\theta = 0.2$ vs. $\theta = 0.6$	0.120	0.962
$\theta = 0.5$ vs. $\theta = 0.6$	0.051	0.488
Panel C: Leader		
	Low R&D Productivity	High R&D Productivity
Alternative Hypothesis:	$Investment_i \neq Investment_j$	
	p-values	p-values
$\theta = 0.1$ vs. $\theta = 0.2$	0.772	0.951
$\theta = 0.1$ vs. $\theta = 0.5$	0.989	0.908
$\theta = 0.1$ vs. $\theta = 0.6$	0.270	0.564
$\theta = 0.2$ vs. $\theta = 0.5$	0.741	0.654
$\theta = 0.2$ vs. $\theta = 0.6$	0.061	0.448
$\theta = 0.5$ vs. $\theta = 0.6$	0.093	0.708

Notes: We utilize the Mann-Whitney-Wilcoxon tests to determine any differences ($i \neq j$) in the distribution of investments across the selected levels of product substitutability for low and high R&D productivity. In Panel A, we report the p-values in the laggard scenario, in Panel B, we report the p-values in the levelled scenario and, in Panel C, we report the p-values in the leader scenario.

H Experimental Instructions

H.1 $K = 2.63, \theta = 0.1, \pi_h = £2.19, \pi_s = £0.91, \pi_l = £0.18$

The purpose of this experimental session is to study how people make decisions in a particular situation. Your earnings will depend upon the decisions you make as well as the decisions that other people make. At the end of the session, you will be paid in cash your total earnings. None of the other participants will be informed of your earnings, and likewise you will not be informed of the earnings of others. Given that nobody will know of each other's identity, all the decisions you make during the experimental session will be anonymous.

For your participation in the experimental session, you will receive an initial payment of £5.

The instructions are simple. If you have a question, please raise your hand. Aside from these questions, any communication with other participants or looking at other participants' screens is not permitted and will lead to your immediate exclusion from the experimental session.

The instructions are identical to all participants.

You are matched with another participant. Each participant manages a firm. Thus, there are 2 firms within the industry. **Both your firm and the other firm will make an investment decision in each of 3 starting situations.** The 3 starting situations differ in the relative ranking of the two firms in the point score as follows:

i) Your firm is one point ahead in the point score from the other firm.

ii) The other firm is one point ahead in the point score from your firm.

iii) Both firms have the same number of points in the point score.

To help you decide on the level of investment to undertake in each of the 3 starting situations, information is provided next. This information pertains to the investment levels and their respective probabilities of success and costs. The higher your investment choice, the more likely it is that your firm's investment will be successful and that you will earn one point in the point score. At the same time, a higher investment also leads to higher costs.

Investment levels take values from 0 to 80. On one hand, the investment level choice indicates the probability of success as a percent; that is, an investment level choice of x, reflects a probability of success of $x\%$. On the other hand, the investment level choice determines the cost; specifically, the cost is calculated using the formula $\frac{1}{2.63} * \frac{x\%}{1-x\%}$, where x is the investment level choice.

The Table displays some indicative (integer) investment levels and their respective probabilities of success as well as the costs at each investment level. Recall that the cost is calculated using the formula $\frac{1}{2.63} * \frac{x\%}{1-x\%}$, where x is the investment level choice. The cost is displayed in 4 decimal points.

After each firm has chosen its investment level in each of the 3 strarting situations, there will be a computer draw that will determine whether the firm's investment in each starting situation is successful. Specifically, the computer will draw an integer from 1 to 100 (all inclusive), where each integer has the same probability of being drawn. If the firm's investment choice in some starting situation is x, hence the probability of success is $x\%$, and the computer draws a number above x, then the firm's investment in that starting situation is

Investment Level	Probability of Success (%)	Cost (£)
0	0	0.0000
2	2	0.0078
4	4	0.0158
6	6	0.0243
8	8	0.0330
10	10	0.0422
12	12	0.0518
14	14	0.0619
16	16	0.0724
18	18	0.0834
20	20	0.0950
22	22	0.1072
24	24	0.1200
26	26	0.1335
28	28	0.1478
30	30	0.1629
32	32	0.1788
34	34	0.1958
36	36	0.2138
38	38	0.2329
40	40	0.2533
42	42	0.2752
44	44	0.2986
46	46	0.3237
48	48	0.3508
50	50	0.3800
52	52	0.4117
54	54	0.4461
56	56	0.4836
58	58	0.5248
60	60	0.5700
62	62	0.6200
64	64	0.6756
66	66	0.7376
68	68	0.8075
70	70	0.8867
72	72	0.9771
74	74	1.0815
76	76	1.2033
78	78	1.3473
80	80	1.5200

unsuccessful. However, if the computer draws a number below or equal to x, then the firm's investment in that starting situation is successful. Note that it is possible that in a starting situation, the investment of a firm is successful, whereas in another starting situation, the investment of that firm is unsuccessful as it all depends on the firm's investment choice in each starting situation and the computer draw about the success (or not) of your investment.

Take starting situation i) where your firm is one point ahead in the point score from the other firm. If your investment is successful, regardless of whether or not the investment of the other firm turns out to be successful, you will again be ahead in the point score by one point. If your investment is unsuccessful and so is the other firm's investment, then again you will be ahead in the point score by one point. If your investment is unsuccessful and the other firm's investment is successful, then you will be tied in the point score. Thus, unless your investment is unsuccessful and the other firm's investment is successful, you will be ahead of the other firm in the point score by one point.

Take starting situation ii) where the other firm is one point ahead in the point score from your firm. If the other firm's investment is successful, regardless of whether or not your investment turns out to be successful, the other firm will again be ahead in the point score by one point. If the other firm's investment is unsuccessful and so is your firm's investment, then again the other firm will be ahead in the point score by one point. If the other firm's investment is unsuccessful and your firm's investment is successful, then you will be tied in the point score. Thus, unless the other firm's investment is unsuccessful and your firm's investment is successful, the other firm will be ahead of your firm in the point score by one point.

Take starting situation iii) where both firms have the same number of points in the point score. If the two firms' investments are both successful or both unsuccessful, then the two firms will still be tied in the point score. The only way for one firm to be ahead in the point score from the other firm is for that firm's investment choice to turn out successful and for the other firm's investment choice to turn out unsuccessful.

Determination of Payoffs

No feedback will be provided until both of you have made all your investment choices in the three starting situations. To determine your payoffs, one starting situation will be selected at random (i.e. each starting situation is equally likely to be drawn).

Should the starting situation i) get chosen (i.e. your firm is one point ahead in the point score from the other firm), your payoffs will be determined based on your investment choice in i), the other participant's investment choice in ii), and the respective outcomes of the two firms' investment decisions, which will determine the **final** relative standing in the point score.

Should the starting situation ii) get chosen (i.e. the other firm is one point ahead in the point score from your firm), your payoffs will be determined based on your investment choice in ii), the other participant's investment choice in i), and the respective outcomes of the two firms' investment decisions, which will determine the **final** relative standing in the point score.

Should the starting situation iii) get chosen (i.e. both firms have the same number of points in the point score), your payoffs will be determined based on your investment choice in iii), the other participant's investment choice in iii), and the respective outcomes of the two firms' investment decisions, which will determine the **final** relative standing in the point score.

The payoffs of each firm depend on their **final** relative standing in the point score after the computer draw about the success (or not) of their investment and the cost of their chosen investment level.

- If your firm is ahead in the point score, then your firm will receive £2.19 minus the cost of your chosen investment level, and the other firm will receive £0.18 minus the cost of its chosen investment level.

- If the other firm is ahead in the point score, then the other firm will receive £2.19 minus the cost of its chosen investment level, and your firm will receive £0.18 minus the cost of your chosen investment level.

- If both firms have the same number of points in the point score, then both your firm and the other firm will receive £0.91 minus the cost of the chosen investment level of each firm.

Your total earnings right now are set at £5. Should you make a loss, this will be deducted from your total earnings of £5.

Examples

Provide your answers to 2 decimal points.

1. Suppose you choose an investment level of 4. What is your probability of success? 0.04 What is your cost? £0.02 Suppose that the other firm chooses an investment level of 20. What is its probability of success? 0.20 What is its cost? £0.10

2. Suppose you choose an investment level of 26. What is your probability of success? 0.26 What is your cost? £0.13 Suppose that the other firm chooses an investment level of 32. What is its probability of success? 0.32 What is its cost? £0.18

3. Suppose you choose an investment level of 28. What is your probability of success? 0.28 What is your cost? £0.15 Suppose that the other firm chooses an investment level of 40. What is its probability of success? 0.40 What is its cost? £0.25

4. Suppose your firm is one point behind in the point score from the other firm, and you choose an investment level of 42. What is your probability of success? 0.42 What is your cost? £0.28 Suppose that the other firm, which is one point ahead in the point score, chooses an investment level of 48. What is its probability of success? 0.48 What is its cost? £0.35 Suppose your investment turns out to be successful, and the other firm's investment turns out to be successful. What are your payoffs? £0.18 - £0.28 = -£0.10 What are your total earnings? £5 - £0.10 = £4.90 What are the payoffs of the other firm? £2.19 - £0.35 = £1.84 What are the total earnings of the other firm? £5 + £1.84 = £6.84

5. Suppose both your firm and the other firm have the same number of points in the point score, and you choose an investment level of 54. What is your probability of success? 0.54 What is your cost? £0.45 Suppose that the other firm chooses an investment level of 40. What is its probability of success? 0.40 What is its cost? £0.25 Suppose your investment turns out to be successful and so does the other firm's investment. What are your payoffs? £0.91 - £0.45 = £0.46 What are your total earnings? £5 + £0.46 = £5.46 What are the payoffs of the other firm? £0.91 - £0.25 = £0.66 What are the total earnings of the other firm? £5 + £0.66 = £5.66

6. Suppose both your firm and the other firm have the same number of points in the point score, and you choose an investment level of 68. What is your probability of success? 0.68 What is your cost? £0.81 Suppose that the other firm, which also has the same number of points, chooses an investment level of 52. What is its probability of success? 0.52 What is its cost? £0.41 Suppose your investment turns out to be unsuccessful, whereas the other firm's investment turns out to be successful. What are your payoffs? £0.18 - £0.81 = -£0.63 What are your total earnings? £5 - £0.63 = £4.37 What are the payoffs of the other firm? £2.19 - £0.41 = £1.78 What are the total earnings of the other firm? £5 + £1.78 = £6.78

7. Suppose both your firm and the other firm have the same number of points in the point score, and you choose an investment level of 56. What is your probability of success? 0.56 What is your cost? £0.48 Suppose that the other firm, which also has the same number of points, chooses an investment level of 8. What is its probability of success? 0.08 What is its cost? £0.03 Suppose your investment turns out to be unsuccessful and so does the other firm's investment. What are your payoffs? £0.91 - £0.48 = £0.43 What are your total earnings? £5 + £0.43 = £5.43 What are the payoffs of the other firm? £0.91 - £0.03 = £0.88 What are the total earnings of the other firm? £5 + £0.88 = £5.88

Quiz

Provide your answers to 2 decimal points.

1. How many firms are within an industry? 2

2. What are your total earnings right now? £5

3. How many investment decisions you need to make? 3

4. If your investment is successful, how many points in the point score do you earn? 1

5. If you choose an investment level of 12, what is your probability of success? 0.12

6. If you choose an investment level of 20, what is your cost? £0.10

7. Suppose you chose an investment level of 20 in the selected starting situation. To determine whether the firm's investment in the selected starting situation is successful the computer draws integer 27. Is the firm's investment in the selected starting situation successful? No

8. Suppose you chose an investment level of 20 in the selected starting situation. To determine whether the firm's investment in the selected starting situation is successful the computer draws integer 17. Is the firm's investment in the selected starting situation successful? Yes

9. Suppose you chose an investment level of 20 in the selected starting situation. To determine whether the firm's investment in the selected starting situation is successful the computer draws integer 20. Is the firm's investment in the selected starting situation successful? Yes

10. Suppose your firm is one point ahead in the point score from the other firm, and you choose an investment level of 30. Your investment turns out to be successful. What are your payoffs? £2.03

11. Suppose your firm is one point ahead in the point score from the other firm, and you choose an investment level of 40. Your investment turns out to be unsuccessful. The other firm's investment turns out to be successful. What are your payoffs? £0.66

12. Suppose the other firm is one point ahead in the point score from your firm, and you choose an investment level of 56. Your investment turns out to be successful. The other firm's investment turns out to be also successful. What are your payoffs? -£0.30

13. Suppose the other firm is one point ahead in the point score from your firm, and you choose an investment level of 26. Your investment turns out to be unsuccessful. What are your payoffs? £0.05

14. Suppose the other firm is one point ahead in the point score from your firm, and you choose an investment level of 26. Your investment turns out to be unsuccessful. What are your total earnings? £5.05

15. Suppose both your firm and the other firm have the same number of points in the point score, and you choose an investment level of 36. Your investment turns out to be unsuccessful. The other firm's investment turns out to be successful. What are your payoffs? -£0.03

16. Suppose both your firm and the other firm have the same number of points in the point score, and you choose an investment level of 70. Your investment turns out to be unsuccessful. The other firm's investment turns out to be also unsuccessful. What are your payoffs? £0.02

17. Suppose both your firm and the other firm have the same number of points in the point score, and you choose an investment level of 44. Your investment turns out to be unsuccessful. The other firm's investment turns out to be also unsuccessful. What are your payoffs? £0.61

18. Suppose both your firm and the other firm have the same number of points in the point score, and you choose an investment level of 20. Your investment turns out to be successful. The other firm's investment turns out to be also successful. What are your payoffs? £0.81

19. Suppose both your firm and the other firm have the same number of points in the point score, and you choose an investment level of 40. Your investment turns out to be successful. The other firm's investment turns out to be unsuccessful. What are your payoffs? £1.94

20. Suppose both your firm and the other firm have the same number of points in the point score, and you choose an investment level of 40. Your investment turns out to be successful. The other firm's investment turns out to be unsuccessful. What are your total earnings? £6.94

Game-Play Stage

You will be asked next to make an investment decision in **each** of 3 starting situations. Remember that the 3 starting situations differ in the relative ranking of the two firms in the point score as follows:

i) Your firm is one point ahead in the point score from the other firm.

ii) The other firm is one point ahead in the point score from your firm.

iii) Both firms have the same number of points in the point score.

The starting situations will be shown to you in no particular order. Recall that once you make all three investment decisions, one starting situation will be selected at random (i.e. each starting situation is equally likely to be drawn).

Once you enter your investment choice, you will be asked to confirm it. You are allowed to enter any investment level choice as long as it spans from 0 to 80 all inclusive. Please note that once you confirm your investment choice, you will not be allowed to change it; that is, your investment choice will be final.

Assume this is starting situation i) where you are asked to make an investment decision knowing that:

i) Your firm is one point ahead in the point score from the other firm.

The Table displays some indicative (integer) investment levels and their respective probabilities of success as well as the costs at each investment level. Recall that the cost is calculated using the formula $\frac{1}{2.63} * \frac{x\%}{1-x\%}$, where x is the investment level choice. The cost is displayed in 4 decimal points.

Remember that:

The payoffs of each firm depend on their **final** relative standing in the point score after the computer draw about the success (or not) of their investment and the cost of their chosen investment level.

If your firm is ahead in the point score, then your firm will receive £2.19 minus the cost of your chosen investment level, and the other firm will receive £0.18 minus the cost of its chosen investment level.

If both firms have the same number of points in the point score, then both your firm and the other firm will receive £0.91 minus the cost of the chosen investment level of each firm.

Once you enter your investment choice, you will be asked to confirm it. You are allowed to enter any investment level choice as long as it spans from 0 to 80 all inclusive. Please note that once you confirm your investment choice, you will not be allowed to change it; that is, your investment choice will be final.

Your firm is one point ahead in the point score from the other firm. What is your investment level?

Investment Level	Probability of Success (%)	Cost (£)
0	0	0.0000
2	2	0.0078
4	4	0.0158
6	6	0.0243
8	8	0.0330
10	10	0.0422
12	12	0.0518
14	14	0.0619
16	16	0.0724
18	18	0.0834
20	20	0.0950
22	22	0.1072
24	24	0.1200
26	26	0.1335
28	28	0.1478
30	30	0.1629
32	32	0.1788
34	34	0.1958
36	36	0.2138
38	38	0.2329
40	40	0.2533
42	42	0.2752
44	44	0.2986
46	46	0.3237
48	48	0.3508
50	50	0.3800
52	52	0.4117
54	54	0.4461
56	56	0.4836
58	58	0.5248
60	60	0.5700
62	62	0.6200
64	64	0.6756
66	66	0.7376
68	68	0.8075
70	70	0.8867
72	72	0.9771
74	74	1.0815
76	76	1.2033
78	78	1.3473
80	80	1.5200

Assume this is starting situation ii) where you are asked to make an investment decision knowing that:

ii) The other firm is one point ahead in the point score from your firm.

The Table displays some indicative (integer) investment levels and their respective probabilities of success as well as the costs at each investment level. Recall that the cost is calculated using the formula $\frac{1}{2.63} * \frac{x\%}{1-x\%}$, where x is the investment level choice. The cost is displayed in 4 decimal points.

Remember that:

The payoffs of each firm depend on their final relative standing in the point score after the computer draw about the success (or not) of their investment and the cost of their chosen investment level.

If the other firm is ahead in the point score, then the other firm will receive £2.19 minus the cost of its chosen investment level, and your firm will receive £0.18 minus the cost of your chosen investment level.

If both firms have the same number of points in the point score, then both your firm and the other firm will receive £0.91 minus the cost of the chosen investment level of each firm.

Once you enter your investment choice, you will be asked to confirm it. You are allowed to enter any investment level choice as long as it spans from 0 to 80 all inclusive. Please note that once you confirm your investment choice, you will not be allowed to change it; that is, your investment choice will be final.

The other firm is one point ahead in the point score from your firm. What is your investment level?

Investment Level	Probability of Success (%)	Cost (£)
0	0	0.0000
2	2	0.0078
4	4	0.0158
6	6	0.0243
8	8	0.0330
10	10	0.0422
12	12	0.0518
14	14	0.0619
16	16	0.0724
18	18	0.0834
20	20	0.0950
22	22	0.1072
24	24	0.1200
26	26	0.1335
28	28	0.1478
30	30	0.1629
32	32	0.1788
34	34	0.1958
36	36	0.2138
38	38	0.2329
40	40	0.2533
42	42	0.2752
44	44	0.2986
46	46	0.3237
48	48	0.3508
50	50	0.3800
52	52	0.4117
54	54	0.4461
56	56	0.4836
58	58	0.5248
60	60	0.5700
62	62	0.6200
64	64	0.6756
66	66	0.7376
68	68	0.8075
70	70	0.8867
72	72	0.9771
74	74	1.0815
76	76	1.2033
78	78	1.3473
80	80	1.5200

Assume this is starting situation iii) where you are asked to make an investment decision knowing that:

iii) Both firms have the same number of points in the point score.

The Table displays some indicative (integer) investment levels and their respective probabilities of success as well as the costs at each investment level. Recall that the cost is calculated using the formula $\frac{1}{2.63} * \frac{x\%}{1-x\%}$, where x is the investment level choice. The cost is displayed in 4 decimal points.

Remember that:

The payoffs of each firm depend on their final relative standing in the point score after the computer draw about the success (or not) of their investment and the cost of their chosen investment level.

If your firm is ahead in the point score, then your firm will receive £2.19 minus the cost of your chosen investment level, and the other firm will receive £0.18 minus the cost of its chosen investment level.

If the other firm is ahead in the point score, then the other firm will receive £2.19 minus the cost of its chosen investment level, and your firm will receive £0.18 minus the cost of your chosen investment level.

If both firms have the same number of points in the point score, then both your firm and the other firm will receive £0.91 minus the cost of the chosen investment level of each firm.

Once you enter your investment choice, you will be asked to confirm it. You are allowed to enter any investment level choice as long as it spans from 0 to 80 all inclusive. Please note that once you confirm your investment choice, you will not be allowed to change it; that is, your investment choice will be final.

Both firms are tied in the point score. What is your investment level?

Investment Level	Probability of Success (%)	Cost (£)
0	0	0.0000
2	2	0.0078
4	4	0.0158
6	6	0.0243
8	8	0.0330
10	10	0.0422
12	12	0.0518
14	14	0.0619
16	16	0.0724
18	18	0.0834
20	20	0.0950
22	22	0.1072
24	24	0.1200
26	26	0.1335
28	28	0.1478
30	30	0.1629
32	32	0.1788
34	34	0.1958
36	36	0.2138
38	38	0.2329
40	40	0.2533
42	42	0.2752
44	44	0.2986
46	46	0.3237
48	48	0.3508
50	50	0.3800
52	52	0.4117
54	54	0.4461
56	56	0.4836
58	58	0.5248
60	60	0.5700
62	62	0.6200
64	64	0.6756
66	66	0.7376
68	68	0.8075
70	70	0.8867
72	72	0.9771
74	74	1.0815
76	76	1.2033
78	78	1.3473
80	80	1.5200