# Machine Games:
# Theory and Experimental Evidence

## Supplementary Appendix

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## Contents

# 1 Theory

## 1.1 Payoff Matrix for Small $\epsilon$

For expositional purposes, in Table 1, we provide the payoff matrix for 'Always-Defect' (ALLD), 'Tit-For-Tat' (TFT), 'Grim-Trigger' (GT) and 'Win-Stay, Lose-Shift' (WSLS) under the assumption of a *small* implementation error level affecting the interaction. We focus on these specific rules for their popularity and performance. Dal Bó and Fréchette (2018) indicate that ALLD, TFT and GT can account for at least 70% of the metadata collected in the Prisoner's Dilemma (PD) games. Furthermore, the popularity of these three seems to extend to more complex PD games with perfect monitoring (see Rand, Fudenberg, and Dreber (2015) and Aoyagi, Bhaskar, and Fréchette (2019)). WSLS is another interesting rule that has been documented to outperform the popular TFT (Nowak and Sigmund (1993)) in the simulations of Axelrod (Axelrod (1984)). We thus also include it in this exposition.

## Table 1: Payoff Matrix With Small Implementation Error Levels

| | | Column Player | | | |
|---|---|---|---|---|---|
| | | $S^{ALLD}$ | $S^{TFT}$ | $S^{GT}$ | $S^{WSLS}$ |
| Row Player | $S^{ALLD}$ | $[\frac{\epsilon^2}{(\epsilon-1)^2}R + \frac{-\epsilon}{(\epsilon-1)}S + \frac{-\epsilon}{(\epsilon-1)}T + P]/[\frac{1}{(\epsilon-1)^2}]$ | $[\frac{2\epsilon^2}{2\epsilon^2-2\epsilon+1}R + \frac{-\epsilon}{\epsilon-1}S + \frac{-2\epsilon^2+2\epsilon}{2\epsilon^2-2\epsilon+1}T + P]/[\frac{-1}{(2\epsilon^2-2\epsilon+1)(\epsilon-1)}]$ | $[\frac{-\epsilon^2}{(\epsilon-1)(2\epsilon^2-2\epsilon+1)}R + \frac{-\epsilon}{(\epsilon-1)}S + \frac{\epsilon}{2\epsilon^2-2\epsilon+1}T + P]/[\frac{-2\epsilon^2+\epsilon-1}{(\epsilon-1)(2\epsilon^2-2\epsilon+1)}]$ | $[\frac{-\epsilon}{\epsilon-1}R + \frac{-\epsilon}{\epsilon-1}S + T + P]/[\frac{-2}{\epsilon-1}]$ |
| | $S^{TFT}$ | $[\frac{2\epsilon^2}{2\epsilon^2-2\epsilon+1}R + \frac{-2\epsilon^2+2\epsilon}{2\epsilon^2-2\epsilon+1}S - \frac{\epsilon}{\epsilon-1}T + P]/[\frac{-1}{(2\epsilon^2-2\epsilon+1)(\epsilon-1)}]$ | $\frac{R+S+T+P}{4}$ | $[\frac{\epsilon}{2\epsilon^2-2\epsilon+1}R + \frac{-2\epsilon^2+2\epsilon}{2\epsilon^2-2\epsilon+1}S + \frac{\epsilon}{2\epsilon^2-2\epsilon+1}T + P]/[\frac{2\epsilon+1}{2\epsilon^2-2\epsilon+1}]$ | $\frac{R+S+T+P}{4}$ |
| | $S^{GT}$ | $[\frac{-\epsilon^2}{(\epsilon-1)(2\epsilon^2-2\epsilon+1)}R + \frac{\epsilon}{2\epsilon^2-2\epsilon+1}S + \frac{-\epsilon}{(\epsilon-1)}T + P]/[\frac{-2\epsilon^2+\epsilon-1}{(\epsilon-1)(2\epsilon^2-2\epsilon+1)}]$ | $[\frac{\epsilon}{2\epsilon^2-2\epsilon+1}R + \frac{\epsilon}{2\epsilon^2-2\epsilon+1}S + \frac{-2\epsilon^2+2\epsilon}{2\epsilon^2-2\epsilon+1}T + P]/[\frac{2\epsilon+1}{2\epsilon^2-2\epsilon+1}]$ | $[\frac{\epsilon}{4\epsilon^2-5\epsilon+2}R + \frac{-2\epsilon^2+2\epsilon}{4\epsilon^2-5\epsilon+2}S + \frac{-2\epsilon^2+2\epsilon}{4\epsilon^2-5\epsilon+2}T + P]/[\frac{2}{4\epsilon^2-5\epsilon+2}]$ | $[\frac{2\epsilon^2-2\epsilon+1}{4\epsilon^2-5\epsilon+2}R + \frac{4\epsilon^3-6\epsilon^2+3\epsilon}{4\epsilon^2-5\epsilon+2}S + \frac{(-2\epsilon+2)(2\epsilon^2-2\epsilon+1)}{4\epsilon^2-5\epsilon+2}T + P]/[\frac{8\epsilon^2-10\epsilon+5}{4\epsilon^2-5\epsilon+2}]$ |
| | $S^{WSLS}$ | $[\frac{-\epsilon}{\epsilon-1}R + S + \frac{-\epsilon}{\epsilon-1}T + P]/[\frac{-2}{\epsilon-1}]$ | $\frac{R+S+T+P}{4}$ | $[\frac{2\epsilon^2-2\epsilon+1}{4\epsilon^2-5\epsilon+2}R + \frac{(-2\epsilon+2)(2\epsilon^2-2\epsilon+1)}{4\epsilon^2-5\epsilon+2}S + \frac{4\epsilon^3-6\epsilon^2+3\epsilon}{4\epsilon^2-5\epsilon+2}T + P]/[\frac{8\epsilon^2-10\epsilon+5}{4\epsilon^2-5\epsilon+2}]$ | $(1-4\epsilon)R + \epsilon S + \epsilon T + 2\epsilon P$ |

*Notes:* We provide the payoff matrix in the case of a small implementation error level affecting the interaction for $S^{ALLD}$, $S^{TFT}$, $S^{GT}$ and $S^{WSLS}$. The payoffs are those of the row player when paired with a column player. We replaced the actions profiles from Subsection 3.4 with the letters in the generic Prisoner's Dilemma; that is, $(A, A)$ is the *Reward* payoff $(R)$, $(A, B)$ is the *Sucker's* payoff (S), $(B, A)$ is the *Temptation* payoff (T) and $(B, B)$ is the *Punishment* payoff (P).

## 1.2 Payoff Matrix for $\epsilon \to 0$

The 16×16 payoff matrix in the case of $\epsilon \to 0$ is provided in the next page.

Table 2: Payoff Matrix Where $\epsilon \to 0$

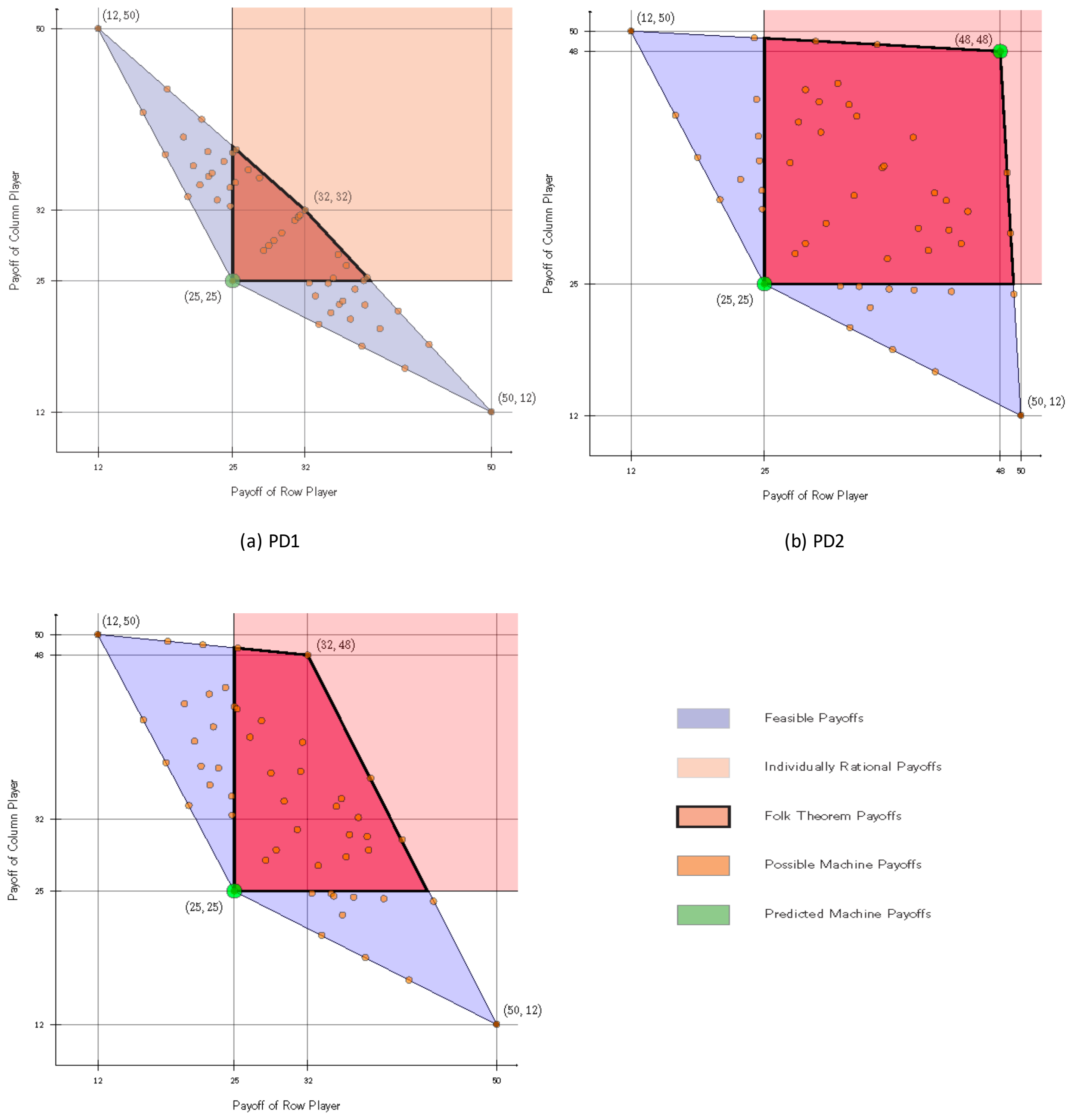
| Row Player | Column Player | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $S^{0000}$ | $S^{0001}$ | $S^{0010}$ | $S^{0011}$ | $S^{0100}$ | $S^{0101}$ | $S^{0110}$ | $S^{0111}$ | $S^{1000}$ | $S^{1001}$ | $S^{1010}$ | $S^{1011}$ | $S^{1100}$ | $S^{1101}$ | $S^{1110}$ | $S^{1111}$ |
| $S^{0000}$ | $P$ | $\frac{T+P}{2}$ | $P$ | $\frac{T+P}{2}$ | $\frac{T+2P}{3}$ | $T$ | $\frac{T+P}{2}$ | $T$ | $P$ | $\frac{T+P}{2}$ | $P$ | $\frac{T+P}{2}$ | $\frac{T+P}{2}$ | $T$ | $\frac{2T+P}{3}$ | $T$ |
| $S^{0001}$ | $\frac{S+P}{2}$ | $\frac{R+P}{2}$ | $\frac{S+T+P}{3}$ | $\frac{R+P}{2}$ | $\frac{2S+T+2P}{5}$ | $\frac{R+T+P}{3}$ | $T$ | $\frac{R+2T+P}{4}$ | $\frac{S+P}{2}$ | $\frac{R+T+P}{3}$ | $\frac{S+T+P}{3}$ | $\frac{R+T+P}{3}$ | $\frac{S+2T+P}{4}$ | $T$ | $T$ | $T$ |
| $S^{0010}$ | $P$ | $\frac{S+T+P}{3}$ | $\frac{S+T+2P}{4}$ | $\frac{S+T}{2}$ | $P$ | $\frac{R+T+P}{3}$ | $P$ | $\frac{R+T+P}{3}$ | $P$ | $\frac{S+T+P}{3}$ | $\frac{S+T+P}{3}$ | $\frac{S+T}{2}$ | $\frac{R+T+2P}{4}$ | $\frac{R+T}{2}$ | $\frac{2R+2T+P}{5}$ | $\frac{R+T}{2}$ |
| $S^{0011}$ | $\frac{S+P}{2}$ | $\frac{R+P}{2}$ | $\frac{S+T}{2}$ | $\frac{R+S+T+P}{4}$ | $\frac{S+P}{2}$ | $\frac{R+P}{2}$ | $\frac{R+S+T+P}{4}$ | $\frac{R+P}{2}$ | $\frac{S+P}{2}$ | $\frac{R+S+T+P}{4}$ | $\frac{S+T}{2}$ | $\frac{S+T}{2}$ | $\frac{R+S+T+P}{4}$ | $\frac{R+T}{2}$ | $\frac{R+T}{2}$ | $\frac{R+T}{2}$ |
| $S^{0100}$ | $\frac{S+2P}{3}$ | $\frac{S+2T+2P}{5}$ | $P$ | $\frac{T+P}{2}$ | $\frac{S+T+2P}{4}$ | $T$ | $\frac{T+2P}{3}$ | $T$ | $\frac{S+2P}{3}$ | $\frac{S+2T+2P}{5}$ | $P$ | $\frac{T+P}{2}$ | $\frac{S+3T+2P}{6}$ | $T$ | $\frac{2T+P}{3}$ | $T$ |
| $S^{0101}$ | $S$ | $\frac{R+S+P}{3}$ | $\frac{R+S+P}{3}$ | $\frac{R+P}{2}$ | $S$ | $\frac{R+S+T+P}{4}$ | $\frac{R+S+T+P}{4}$ | $\frac{R+T+P}{3}$ | $S$ | $\frac{R+S+T+P}{4}$ | $\frac{R+S+T+P}{4}$ | $\frac{R+T+P}{3}$ | $\frac{S+T}{2}$ | $T$ | $T$ | $T$ |
| $S^{0110}$ | $\frac{S+P}{2}$ | $S$ | $P$ | $\frac{R+S+T+P}{4}$ | $\frac{S+2P}{3}$ | $\frac{R+S+T+P}{4}$ | $P$ | $\frac{R+T+P}{3}$ | $\frac{2S+P}{3}$ | $S$ | $\frac{R+S+T+P}{4}$ | $\frac{R+S+T}{3}$ | $\frac{R+S+T+P}{4}$ | $\frac{2R+S+2T}{5}$ | $\frac{2R+2T+P}{5}$ | $\frac{R+T}{2}$ |
| $S^{0111}$ | $S$ | $\frac{R+2S+P}{4}$ | $\frac{R+S+P}{3}$ | $\frac{R+P}{2}$ | $S$ | $\frac{R+S+P}{3}$ | $\frac{R+S+P}{3}$ | $\frac{R+P}{2}$ | $S$ | $S$ | $\frac{R+S+T}{3}$ | $\frac{R+S+T}{3}$ | $\frac{R+2S+T}{4}$ | $\frac{2R+S+2T}{5}$ | $\frac{R+T}{2}$ | $\frac{R+T}{2}$ |
| $S^{1000}$ | $P$ | $\frac{T+P}{2}$ | $P$ | $\frac{T+P}{2}$ | $\frac{T+2P}{3}$ | $T$ | $\frac{2T+P}{3}$ | $T$ | $P$ | $\frac{R+2T+2P}{5}$ | $P$ | $\frac{R+2T+2P}{5}$ | $\frac{R+2T+3P}{6}$ | $\frac{R+2T}{3}$ | $\frac{R+2T+P}{4}$ | $\frac{R+2T}{3}$ |
| $S^{1001}$ | $\frac{S+P}{2}$ | $\frac{R+S+P}{3}$ | $\frac{S+T+P}{3}$ | $\frac{R+S+T+P}{4}$ | $\frac{2S+T+2P}{5}$ | $\frac{R+S+T+P}{4}$ | $T$ | $T$ | $\frac{R+2S+2P}{5}$ | $R$ | $\frac{R+S+T+P}{4}$ | $R$ | $\frac{R+S+T+P}{4}$ | $\frac{2R+T}{3}$ | $\frac{R+2T}{3}$ | $\frac{R+T}{2}$ |
| $S^{1010}$ | $P$ | $\frac{S+T+P}{3}$ | $\frac{S+T+P}{3}$ | $\frac{S+T}{2}$ | $P$ | $\frac{R+S+T+P}{4}$ | $\frac{R+S+T+P}{4}$ | $\frac{R+S+T}{3}$ | $P$ | $\frac{R+S+T+P}{4}$ | $\frac{R+S+T+P}{4}$ | $\frac{R+S+T}{3}$ | $\frac{R+P}{2}$ | $R$ | $R$ | $R$ |
| $S^{1011}$ | $\frac{S+P}{2}$ | $\frac{R+S+P}{3}$ | $\frac{S+T}{2}$ | $\frac{S+T}{2}$ | $\frac{S+P}{2}$ | $\frac{R+S+P}{3}$ | $\frac{R+S+T}{3}$ | $\frac{R+S+T}{3}$ | $\frac{R+2S+2P}{5}$ | $R$ | $\frac{R+S+T}{3}$ | $\frac{2R+S+T}{4}$ | $\frac{2R+S+P}{4}$ | $R$ | $R$ | $R$ |
| $S^{1100}$ | $\frac{S+P}{2}$ | $\frac{2S+T+P}{4}$ | $\frac{R+S+2P}{4}$ | $\frac{R+S+T+P}{4}$ | $\frac{3S+T+2P}{6}$ | $\frac{S+T}{2}$ | $\frac{R+S+T+P}{4}$ | $\frac{R+S+2T}{4}$ | $\frac{R+2S+3P}{6}$ | $\frac{R+S+T+P}{4}$ | $\frac{R+P}{2}$ | $\frac{2R+T+P}{4}$ | $\frac{R+S+T+P}{4}$ | $\frac{2R+S+3T}{6}$ | $\frac{3R+2T+P}{6}$ | $\frac{R+T}{2}$ |
| $S^{1101}$ | $S$ | $S$ | $\frac{R+S}{2}$ | $\frac{R+S}{2}$ | $S$ | $S$ | $\frac{2R+2S+T}{5}$ | $\frac{2R+2S+T}{5}$ | $\frac{R+2S}{3}$ | $\frac{2R+S}{3}$ | $R$ | $R$ | $\frac{2R+3S+T}{6}$ | $\frac{2R+S+T}{4}$ | $\frac{2R+T}{3}$ | $\frac{2R+T}{3}$ |
| $S^{1110}$ | $\frac{2S+P}{3}$ | $S$ | $\frac{2R+2S+P}{5}$ | $\frac{R+S}{2}$ | $\frac{2S+P}{3}$ | $S$ | $\frac{2R+2S+P}{5}$ | $\frac{R+S}{2}$ | $\frac{R+2S+P}{4}$ | $\frac{R+2S}{3}$ | $R$ | $R$ | $\frac{3R+2S+P}{6}$ | $\frac{2R+S}{3}$ | $R$ | $R$ |
| $S^{1111}$ | $S$ | $S$ | $\frac{R+S}{2}$ | $\frac{R+S}{2}$ | $S$ | $S$ | $\frac{R+S}{2}$ | $\frac{R+S}{2}$ | $\frac{R+2S}{3}$ | $\frac{R+S}{2}$ | $R$ | $R$ | $\frac{R+S}{2}$ | $\frac{2R+S}{3}$ | $R$ | $R$ |

*Notes:* We provide the payoff matrix in the case of $\epsilon \to 0$. Each transition rule is denoted with its respective quadruple but without the commas. The payoffs are those of the row player when paired with a column player. We replaced the actions profiles from Subsection 3.4 with the letters in the generic Prisoner's Dilemma; that is, $(A, A)$ is the *Reward* payoff $(R)$, $(A, B)$ is the *Sucker's* payoff (S), $(B, A)$ is the *Temptation* payoff (T) and $(B, B)$ is the *Punishment* payoff (P). Furthermore, each entry indicates the average frequency spent on $R$, $S$, $T$ and $P$, respectively. For instance, the entry $\frac{2R+S}{3}$ indicates that $\frac{2}{3}$ of the time was spent on regime $R$ and $\frac{1}{3}$ of the time was spent on regime $S$.

## 1.3 Predicted Machine Payoffs vs. Folk Theorem Payoffs

Figures 1-3 provide the predicted machine payoffs based on the equilibrium pairs derived in the Prisoner's Dilemma, Stag Hunt and Battle of the Sexes games, respectively. We contrast the predicted machine payoffs with the equilibrium payoffs of the Folk Theorem. We also provide the possible machine payoffs based on our bound of no more than two states in a machine. In PD1 and APD3, the machine payoffs predicted are $(25, 25)$, whereas in PD2, the predicted machine payoffs are $(25, 25)$ or $(48, 48)$. In SH1, the predicted payoffs are $(40, 40)$ or $(42.5, 42.5)$ or $(45, 45)$, while in SH2, the predicted payoffs are $(12, 12)$ or $(28.5, 28.5)$ or $(45, 45)$. In ASH3, the predicted payoffs are $(40, 12)$ or $(42.5, 28.5)$ or $(45, 45)$. In BoS1, the predicted machine payoffs are $(9, 15)$ or $(15, 9)$ or $(12, 12)$, whereas in ABoS2, the predicted machine payoffs are $(9, 17)$ or $(20, 10)$ or $(14.5, 13.5)$. In contrast to the theoretical predictions in the infinitely-repeated games where the equilibrium set of payoffs is very large, in the proposed framework, the payoff predictions are sharp.

Figure 1: Predicted Machine Payoffs in the Prisoner's Dilemma Games

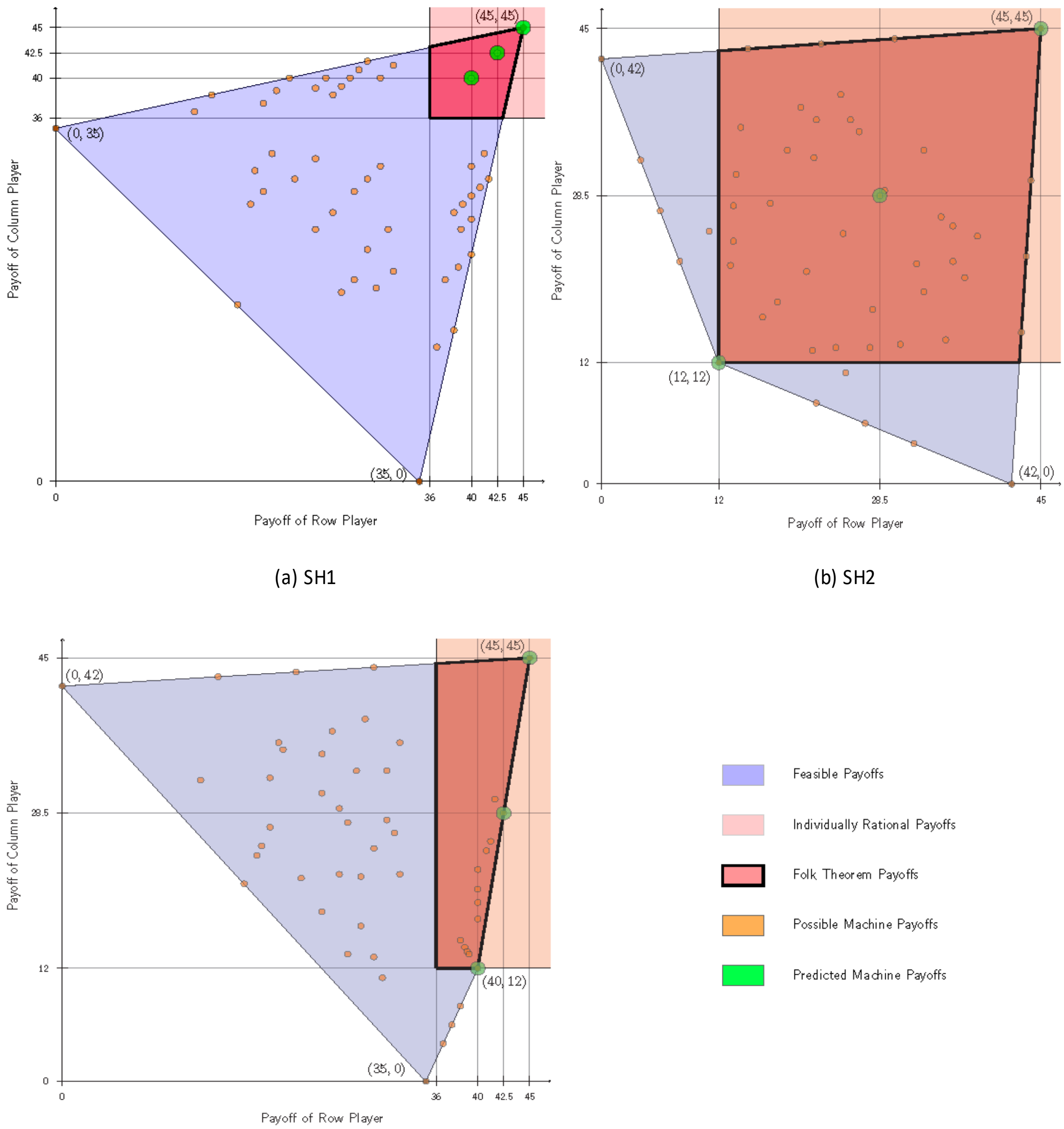



(a) PD1

(b) PD2

(c) APD3

Figure 2: Predicted Machine Payoffs in the Stag Hunt Games

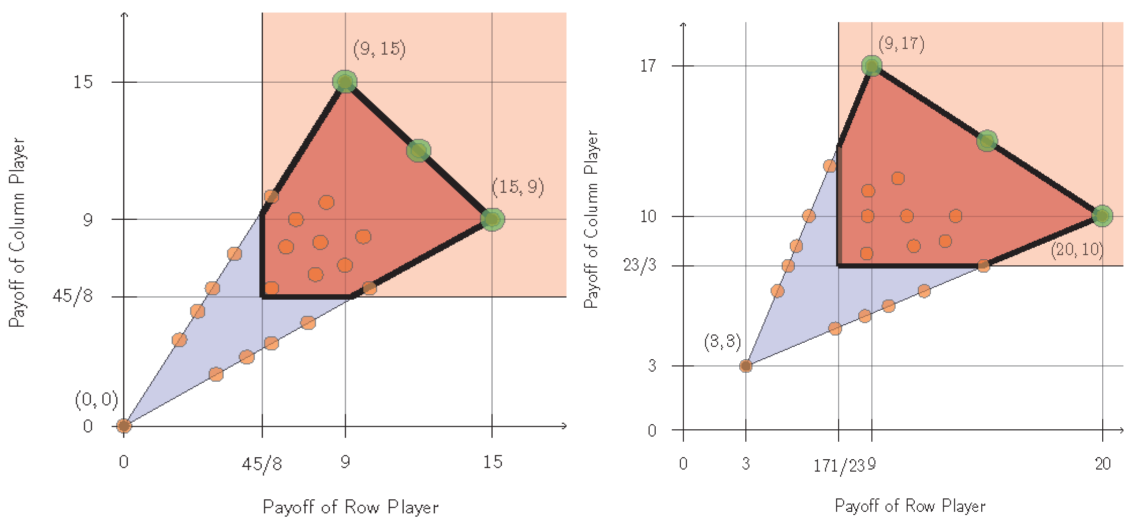



(a) SH1

(b) SH2

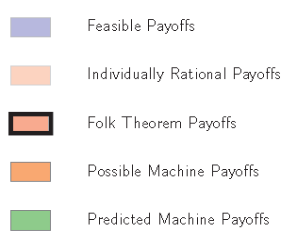
(c) ASH3

Figure 3: Predicted Machine Payoffs in the Battle of the Sexes Games




(a) BoS1

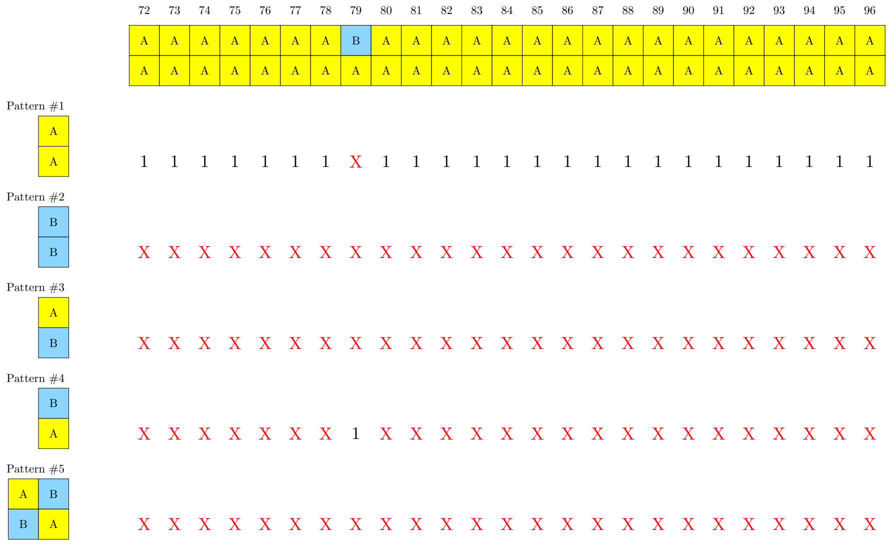
(b) ABoS2



# 2 Examples of the Action-Convergence Criterion

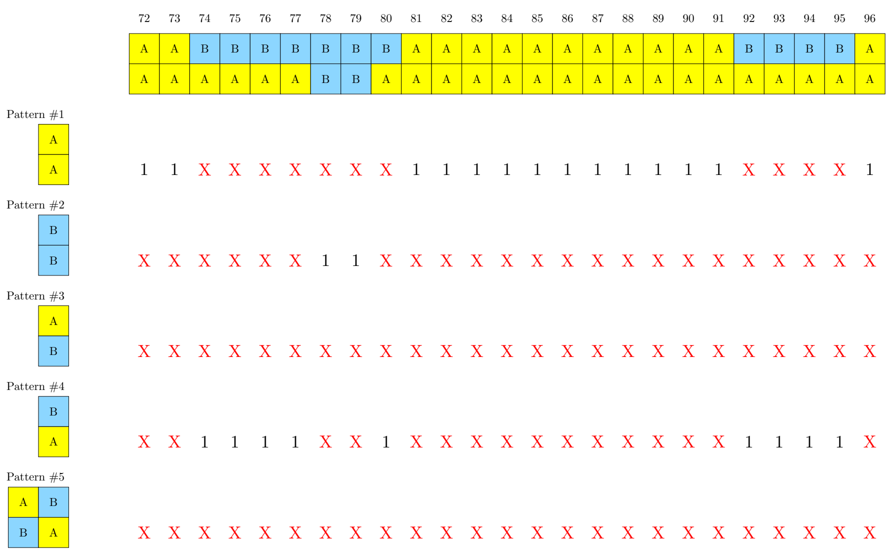
Recall that our pattern mining technique, the action-convergence criterion, is taken from the study of Ioannou et al. (2025). In our manuscript, we focus on (0.9)-convergence; that is, we allow up to two errors within the last 20 periods of game play. In Figures 4-7, we provide expositional examples of the criterion. The figures display sequences in the last 25 periods of game play, though our criterion only considers the last 20 periods. Figures 4 and 5 display sequences from our dataset, whereas Figures 6 and 7 display fictional sequences. Importantly, we provide the matching between the sequences and some selected patterns. The patterns in Figures 4 and 5 are based on the ones of Table 3 in the main text, whereas those in Figures 6 and 7 consist of various ones of different length. In Figure 4, the sequence is taken from APD3 and is (0.9)-convergent whereas, in Figure 5, the sequence is taken from ASH3 and is (0.9)-divergent. In Figure 7, three action profiles in the sequence have been changed, relative to those in Figure 6, in periods 81, 87 and 93 to illustrate that the errors of the longer pattern #5 went down.

Figure 4: Game Play of PairID27 in APD3



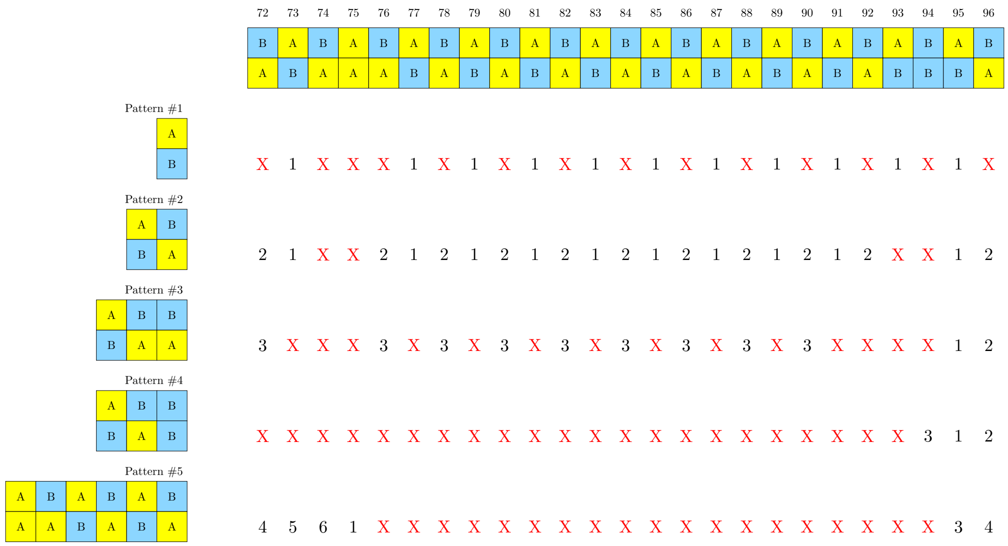

*Notes:* We provide the last 25 periods of game play. Each row contains a pattern (taken from Table 3 in the main text), and the numbers across each pattern indicate which element the pattern should start from to guarantee a match. In case of an error, a red X is inserted at the corresponding period. The sequence of game play is 0.9-convergent to pattern #1.

Figure 5: Game Play of PairID31 in ASH3



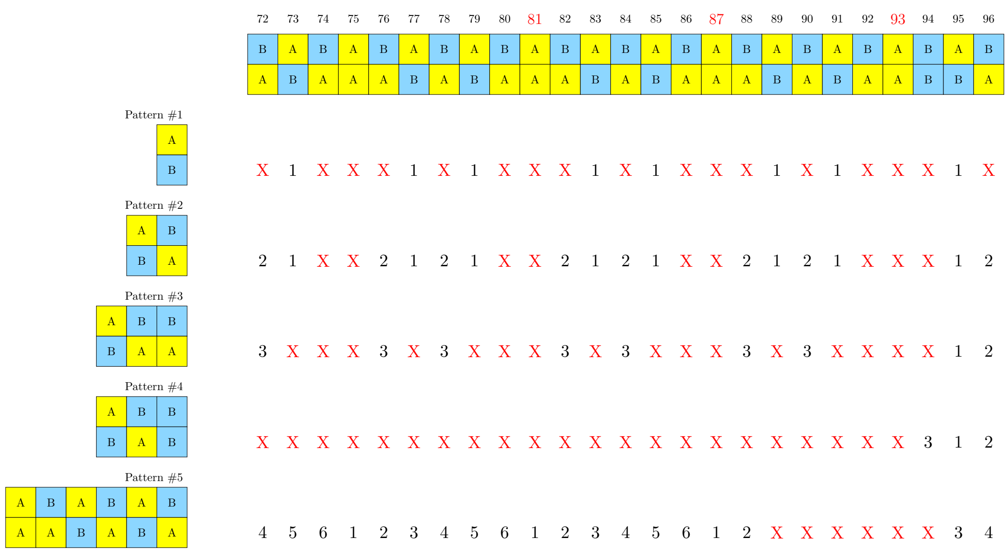

*Notes:* We provide the last 25 periods of game play. Each row contains a pattern (taken from Table 3 in the main text), and the numbers across each pattern indicate which element the pattern should start from to guarantee a match. In case of an error, a red X is inserted at the corresponding period. The sequence of game play is 0.9-divergent.

Figure 6: Fictional Game Play 1




*Notes:* We provide the last 25 periods of some fictional game play. Various patterns of different length are provided. The numbers across each pattern indicate which element the pattern should start from to guarantee a match. In case of an error, a red X is inserted at the corresponding period.

Figure 7: Fictional Game Play 1'




*Notes:* We provide the last 25 periods of some fictional game play. The patterns from Figure 6 are reproduced. The numbers across each pattern indicate which element the pattern should start from to guarantee a match. In case of an error, a red X is inserted at the corresponding period. Three action profiles in the sequence have been changed, relative to those in Figure 6, in periods 81, 87 and 93 to illustrate that the errors of the longer pattern #5 went down.

# 3 Robustness Checks

We provide next our robustness checks on both the number of errors (1, 2 and 3) and intervals of game-play periods (20 and 25 periods) based on the action-convergence criterion. The analysis confirms the insensitivity of the results to these reasonable choices of errors and period intervals.

Table 3: 1 Error & 20 Periods

| | PD1 | PD2 | APD3 | SH1 | SH2 | ASH3 | BoS1 | ABoS2 |
|---|---|---|---|---|---|---|---|---|
| Data Points | 32 | 34 | 35 | 31 | 32 | 33 | 30 | 31 |
| Convergent | 23 | 28 | 21 | 23 | 28 | 25 | 18 | 16 |
| A / A | 3 | 24 | 3 | 16 | 26 | 22 | | |
| B / B | 20 | 4 | 18 | 7 | 2 | 3 | | |
| A / B | | | | | | | 5† | 2 |
| B / A | | | | | | | | 4 |
| A B / B A | | | | | | | 13 | 10 |
| Divergent | 9 | 6 | 14 | 8 | 4 | 8 | 12 | 15 |

Equal Payoffs    Pareto Efficient

*Notes:* The † (next to 5) is to highlight that given that the matrix is symmetric, pattern (A,B) and pattern (B,A) in the following row are interchangeable.

Table 4: 3 Errors & 20 Periods

| | PD1 | PD2 | APD3 | SH1 | SH2 | ASH3 | BoS1 | ABoS2 |
|---|---|---|---|---|---|---|---|---|
| Data Points | 32 | 34 | 35 | 31 | 32 | 33 | 30 | 31 |
| Convergent | 26 | 28 | 22 | 25 | 29 | 26 | 20 | 19 |
| A<br>A | 3 | 24 | 3 | 16 | 26 | 22 | | |
| B<br>B | 23 | 4 | 19 | 9 | 3 | 4 | | |
| A<br>B | | | | | | | 7† | 3 |
| B<br>A | | | | | | | | 6 |
| A B<br>B A | | | | | | | 13 | 10 |
| Divergent | 6 | 6 | 13 | 6 | 3 | 7 | 10 | 15 |

Equal Payoffs    Pareto Efficient

*Notes:* The † (next to 7) is to highlight that given that the matrix is symmetric, pattern (A,B) and pattern (B,A) in the following row are interchangeable.

Table 5: 1 Error & 25 Periods

| | PD1 | PD2 | APD3 | SH1 | SH2 | ASH3 | BoS1 | ABoS2 |
|---|---|---|---|---|---|---|---|---|
| Data Points | 32 | 34 | 35 | 31 | 32 | 33 | 30 | 31 |
| Convergent | 22 | 28 | 18 | 23 | 28 | 24 | 16 | 13 |
| A / A | 3 | 24 | 3 | 16 | 26 | 21 | | |
| B / B | 19 | 4 | 15 | 7 | 2 | 3 | | |
| A / B | | | | | | | 3† | 1 |
| B / A | | | | | | | | 2 |
| A B / B A | | | | | | | 13 | 10 |
| Divergent | 10 | 6 | 17 | 8 | 4 | 9 | 14 | 18 |

Equal Payoffs

Pareto Efficient

*Notes:* The † (next to 3) is to highlight that given that the matrix is symmetric, pattern (A,B) and pattern (B,A) in the following row are interchangeable.

Table 6: 2 Errors & 25 Periods

| | PD1 | PD2 | APD3 | SH1 | SH2 | ASH3 | BoS1 | ABoS2 |
|---|---|---|---|---|---|---|---|---|
| Data Points | 32 | 34 | 35 | 31 | 32 | 33 | 30 | 31 |
| Convergent | 24 | 28 | 19 | 25 | 29 | 26 | 18 | 18 |
| A/A | 3 | 24 | 3 | 16 | 26 | 22 | | |
| B/B | 21 | 4 | 16 | 9 | 3 | 4 | | |
| A/B | | | | | | | 5† | 2 |
| B/A | | | | | | | | 6 |
| A B / B A | | | | | | | 13 | 10 |
| Divergent | 8 | 6 | 16 | 6 | 3 | 7 | 12 | 13 |

Equal Payoffs    Pareto Efficient

*Notes:* The † (next to 5) is to highlight that given that the matrix is symmetric, pattern (A,B) and pattern (B,A) in the following row are interchangeable.

Table 7: 3 Errors & 25 Periods

| | PD1 | PD2 | APD3 | SH1 | SH2 | ASH3 | BoS1 | ABoS2 |
|---|---|---|---|---|---|---|---|---|
| Data Points | 32 | 34 | 35 | 31 | 32 | 33 | 30 | 31 |
| Convergent | 24 | 28 | 20 | 25 | 29 | 26 | 20 | 19 |
| A / A | 3 | 24 | 3 | 16 | 26 | 22 | | |
| B / B | 21 | 4 | 17 | 9 | 3 | 4 | | |
| A / B | | | | | | | 7† | 3 |
| B / A | | | | | | | | 6 |
| A B / B A | | | | | | | 13 | 10 |
| Divergent | 8 | 6 | 15 | 6 | 3 | 7 | 10 | 12 |

Equal Payoffs

Pareto Efficient

*Notes:* The † (next to 7) is to highlight that given that the matrix is symmetric, pattern (A,B) and pattern (B,A) in the following row are interchangeable.

# 4 Experimental Instructions [APD3]

The purpose of this experiment is to study how people make decisions in a particular situation. Your earnings will depend upon the decisions you make as well as the decisions that other people make. At the end of the session, you will be paid in private your total earnings. None of the other participants will be informed of your earnings, and likewise you will not be informed of the earnings of others. The currency used in the experiment is **Experimental Currency Units (ECUs)**. The conversion rate between ECUs and Dollars is 500:1. For your participation in the experiment, you will receive an initial payment of 2,500 ECUs. Based on the conversion rate provided, this amounts to $5.

**The instructions are identical to all participants. After you read the instructions, there will be a Quiz to ensure your understanding of the instructions and procedures.**

You will be asked to make a decision between two choices: X and Y. A payoff table will be given to you at the start of the interaction. **These payoffs will remain the same throughout the interaction.** The following is an example of a payoff table.

For instance, if you choose X and the other participant chooses Y in the period, your payoff will be 30 ECUs in this period, and the other participant's payoff will be 70 ECUs in this period.

| Your choice | Other's choice | Your payoff | Other's payoff |
|---|---|---|---|
| X | X | 60 | 60 |
| X | Y | 30 | 70 |
| Y | X | 70 | 30 |
| Y | Y | 10 | 10 |

As soon as both you and the other participant make a choice, you will be provided with feedback. The feedback consists of your choice, the other participant's choice, your payoff, the other participant's payoff, your cumulative payoff, and the other participant's cumulative payoff.

## Number of Periods

The number of periods you will play in this game is determined as follows. After each period, a random number generator will draw an integer from $\{1, 2, 3, 4, ..., 98, 99, 100\}$ where each integer is equally likely. If the integer drawn is 1, then the game will end. If the number drawn is any integer from $\{2, 3, 4, ..., 98, 99, 100\}$, then the game will continue for one more period. In other words, at the end of each period, there is a 99% chance that the game will continue for one more period.

## Payoffs

Your total earnings in ECUs will be the sum of the payoffs accumulated in each period. The following example provides the interface you will see during the interaction. The history of the interaction is shown at the bottom. To make a choice (X or Y), you need to press the radio button of your choice.

Period 5

| Your choice | Other's choice | Your payoff | Other's payoff |
|---|---|---|---|
| X | X | 60 | 60 |
| X | Y | 30 | 70 |
| Y | X | 70 | 30 |
| Y | Y | 10 | 10 |

Please make your choice

| |
|---|
| X |
| Y |

**History**

| Period | Your choice | Other's choice | Your payoff | Other's payoff | Your cumulative payoff | Other's cumulative payoff |
|---|---|---|---|---|---|---|
| 4 | Y | Y | 10 | 10 | 140 | 180 |
| 3 | X | Y | 30 | 70 | | |
| 2 | X | Y | 30 | 70 | | |
| 1 | Y | X | 70 | 30 | | |

## 4.1 Quiz

Answer the questions based on the following payoff table.

| Your choice | Other's choice | Your payoff | Other's payoff |
|---|---|---|---|
| X | X | 60 | 60 |
| X | Y | 30 | 70 |
| Y | X | 70 | 30 |
| Y | Y | 10 | 10 |

- What is your payoff if you choose Y and the other participant chooses Y? 10
- What is the other participant's payoff if you choose Y and the other participant chooses X? 30
- What is your payoff if you choose Y and the other participant chooses X? 70
- What is the other participant's payoff if you choose Y and the other participant also chooses Y? 10
- Suppose the interaction is currently in period 53; what is the chance that the interaction will end after this period? 1%
- Suppose the interaction is currently in period 71; what is the chance that the interaction will end after this period? 1%

### Instructions for the Match

You have just been matched with another participant. You will interact with the same participant throughout the interaction.

The payoff table is fixed from period to period and is shown below.

| Your choice | Other's choice | Your payoff | Other's payoff |
|---|---|---|---|
| X | X | 32 | 48 |
| X | Y | 12 | 50 |
| Y | X | 50 | 12 |
| Y | Y | 25 | 25 |

The experiment proceeds as fast as the slower participant. Thus, the experiment may take longer than expected if you or the other participant take a long time to make a decision. Recall that at the end of each period, there is a 99% chance that the game will continue for one more period.

# References

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