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Coordination Games: Escaping the Straitjacket

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Abstract

Coordination problems lie at the heart of many real-world financial phenomena. Classic examples are bank runs, debt refinancing, startups under network externalities, and speculative currency attacks. The lack of predictability in such problems poses a serious problem for many academics and practitioners when it comes to predicting, for instance, the onset of speculative currency attacks. Global games and Poisson games have been proposed to address this equilibrium indeterminacy. Global games assume that agents face idiosyncratic uncertainty about economic fundamentals, whereas Poisson games model the number of actual players as a Poisson random variable to capture population uncertainty in large games. We first provide an overview of the literature and propose unresolved questions for future research. Next, we describe the innovative methodology in Ioannou and Makris (2019) that accommodates for the aforementioned sources of uncertainty. We complement existing results on single-shot experiments with new results that pertain to repeated interactions.

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1 Introduction

A plethora of financial phenomena feature strategic complementarities; that is, individual gains from taking a particular action are non-decreasing in the number of agents who chose the same action. Classic examples are bank runs (e.g. Diamond and Dybvig (1983), Milgrom and Roberts (1990), Goldstein and Pauzner (2005)), debt refinancing (e.g. Rochet and Vives (2004), Morris and Shin (2004), Trevino (2020)), startups under network externalities, and speculative currency attacks (e.g. Obstfeld (1996), Morris and Shin (1998)). For instance, a bank run may arise when the number of depositors who withdraw their deposits at the bank is sufficiently high. Along similar lines, an early liquidation may occur when an insufficient number of creditors roll over their loans. In the context of start-up investments and new technology adoption under network externalities, individuals may collectively settle for an inferior commodity simply because they expect sufficiently many other agents to do the same.

The presence of strategic complementarities coupled with the agents' self-fulfilling nature of beliefs,[1] and common knowledge in economic fundamentals and number of stakeholders, result in equilibrium indeterminacy. This lack of predictability poses a serious problem for many academics and practitioners when it comes to predicting, for instance, the onset of speculative currency attacks.[2] The ensuing common view is that in order to escape a prediction of indeterminacy of equilibria, the environment needs to have a sufficiently large degree of heterogeneity and/or of asymmetric information.

Global games, first studied by Carlsson and van Damme (1993), deploy uncertainty about the state of economic fundamentals (for example, the profitability of a successful speculative attack) while maintaining the assumption of common knowledge in the number of stakeholders.[3] Crucially, the Global game prediction about, say, the onset of speculative attacks manifests a threshold level of economic fundamentals that defines two areas in the region where standard (common knowledge) Coordina-

[1] For example, if speculators believe that a currency is not in danger of an attack, their inaction safeguards the currency, while if speculators believe that the attack is eminent, their actions envisaged in those beliefs will precipitate the crisis itself.

[2] Consider the collapse of the Mexican peso in the early 1990s. As Morris and Shin (1998) point out, by most accounts, the currency was susceptible to an attack well before the crisis that led to its eventual devaluation.

[3] See Herrendorf, Valentinyi, and Waldman (2000), Frankel and Pauzner (2000), Burdzy, Frankel, and Pauzner (2001), Dasgupta (2007).

tion games predict multiplicity of equilibria: one in which a successful attack takes place, and another, where a successful attack does not materialize.

An alternative approach to Global games, is motivated instead by the fact that, in the above strategic environments, the number of stakeholders is often very large. As Myerson (2000) points out, in games with a very large number of players, "it is unrealistic to assume that every player knows all the other players in the game; instead, a more realistic model should admit some uncertainty about the number of players in the game" (p. 7). Introducing population uncertainty changes the strategic environment in a fundamental way. In particular, it creates conceptual problems in analyzing the resulting game as a Bayesian game. The reason is that introducing population uncertainty implies that players can no longer assign a strategy to other individual players simply because they are not aware of who they all are.[4] This approach thus models the number of actual players as a Poisson random variable (see Makris (2008)). Importantly, the Poisson game prediction is that, for any given set of economic fundamentals, no speculative attack will take place as long as the ratio of the short-selling cost per reward is greater than the probability of having sufficiently many players in the game; otherwise, multiplicity of equilibrium outcomes still arises.

Motivated by the aforementioned theoretical studies, Ioannou and Makris (2019) utilize an innovative methodology to investigate whether idiosyncratic uncertainty about economic fundamentals or uncertainty about the number of actual players may influence subjects' behavior in single-shot experiments. The set-up and findings of this study are described in Section 3, where we also complement existing results with new (previously unpublished) ones that pertain to repeated interactions. However, before that, we present next an overview of the relevant theoretical and experimental literature as well as propose unresolved questions to direct future research.

2 Theoretical and Experimental Literature

Standard Coordination games with common knowledge of economic fundamentals and number of stakeholders have been studied extensively both theoretically and exper-

[4] A direct implication of this is that the insights of Global games cannot be used directly in this framework, in contrast to what one might have thought by extrapolating the insights of Global games to a set-up where the number of players is treated as yet another uncertain fundamental of the game in hand.

imentally (e.g. Milgrom and Roberts (1990), Van Huyck, Battalio, and Beil (1990), Van Huyck, Battalio, and Beil (1991), Van Huyck, Battalio, and Beil (1993), Hanaki, Sethi, Erev, and Peterhansl (2005), Hyndman, Terracol, and Vaksmann (2009), Ioannou and Romero (2014), Ioannou, Mathevet, and Romero (2022)). Furthermore, Global games have opened up a multitude of interesting avenues of theoretical and experimental investigation (see Table 1). In sharp contrast, Poisson population-uncertainty games have been heavily understudied (see Table 2). This is indeed a big shortcoming, especially in the experimental literature, as most real-world financial applications of Coordination games involve a large but unknown number of stakeholders. Thus, we propose next some promising avenues for future experimental and theoretical work. Poisson population uncertainty takes center stage in this research program.

Given that the motivation behind Poisson population-uncertainty games is to capture the *largeness* of these games, it is imperative to conduct experiments with a sufficiently large number of participants. The added value of such an approach is that it will mimic our motivating examples (e.g. bank runs). The necessity of conducting these experiments over the Internet to preserve the underlying assumption of population uncertainty gives significant leeway to the experimenter over the sample size, type of experiment (e.g. artefactual), geographic location and platform used (e.g. Mechanical Turk, Prolific).

Another fruitful investigation that is open both theoretically and experimentally is to combine uncertainty in economic fundamentals with uncertainty in the number of participants in an effort to capture an environment that is well in line with real-world financial applications. From a theoretical point of view, it would be interesting to see if there is some surprising interaction between the two types of uncertainty, which, for instance, restores equilibrium indeterminacy.[5] Alternatively, perhaps one source of uncertainty dominates the other, hence the respective theoretical prediction becomes binding. From an experimental point of view, it would be important to determine how subjects behave in an environment where uncertainty comes from two distinct sources.

Finally, an important direction for future research would be the provision of a uni-

[5] A set of papers (e.g. Angeletos and Werning (2006), Angeletos, Hellwig, and Pavan (2007)) shows how prices, the action of a policymaker, and past outcomes can function as endogenous public signals that may restore equilibrium multiplicity in settings that would otherwise have exhibited a unique equilibrium.

Table 1: Literature on Global Games

Panel A: *Theoretical*		
Authors' Names	Application	Main Findings
Carlsson and van Damme (1993)	Perturbation of players' payoff information in 2×2 games	Iterated dominance leads to risk-dominant equilibrium
Morris and Shin (1998)	Strategic interaction between government and speculators in foreign exchange market	Unique equilibrium when small noise in private signals of economic fundamentals
Laskar (2013)	Ambiguity-averse creditors decide whether to roll over loans for investment project	Probability of crisis increases as ambiguity reduces coordination
Dahleh, Tahbaz-Salehi Tsitsiklis, and Zoumpoulis (2016)	Local information channels to capture decentralized modes of communication	Set of equilibria highly sensitive to how information is locally shared among agents
Trevino (2020)	Financial contagion where investors receive a noisy private signal on state of economy and decide whether to roll over loans	Correlation between signal's precision and economic fundamentals navigates comparative statics

Panel B: *Experimental*		
Authors' Names	Application	Main Findings
Heinemann, Nagel and Ockenfels (2004)	Speculative attack model of Morris and Shin (1998)	Use of threshold strategies; behavior in Global consistent with respective theory
Cabrales, Nagel and Armenter (2007)	2×2 set-up of Carlsson and van Damme (1993)	Long-run behavior in Global converges to respective theory; short-run behavior between Global prediction and payoff-dominant equilibrium
Cornand (2006)	Speculative attack model of Morris and Shin (1998) with noisy public signal	Probability of attack is smaller and predictability better with private and public signals relative to baseline
Darai, Kogan, Kwasnica and Weber (2017)	Provision of endogenous public aggregate-sentiment signal in addition to private signal	Aggregate-sentiment measure helps coordinate behavior on more efficient equilibria
Szkup and Trevino (2020)	Manipulation of private signal's precision	As signal's noise decreases behavior tends towards efficient threshold

Notes: We provide an overview of the theoretical (in Panel A) and experimental (in Panel B) contributions in the strand of the literature on Global games.

Table 2: Literature on Poisson Games

Panel A: *Theoretical*		
Authors' Names	Application	Main Findings
Myerson (2000)	Mathematical framework for analysis of large games with population uncertainty	Poisson games are uniquely characterized by environmental equivalence
Makris (2008)	Coordination problem modelled as Poisson game	Conditions where equilibrium is unique
Panel B: *Experimental*		
Authors' Names	Application	Main Findings
Ioannou and Makris (2019)	Uncertainty in economic fundamentals vs. uncertainty in number of stakeholders	Population uncertainty has more pronounced impact; also, consistent with respective theory

Notes: We provide an overview of the theoretical (in Panel A) and experimental (in Panel B) contributions in the strand of the literature on Poisson games.

fied theory of explaining behavior across various environments. Such fruitful attempts have been undertaken by Heinemann, Nagel, and Ockenfels (2009), and Kneeland (2016). The former study estimates various parameters of a Global game and shows that the estimated model performs well on that front. The latter study utilizes the experimental dataset of Heinemann, Nagel, and Ockenfels (2004) to calibrate a model that rests on the limited-depth-of-reasoning solution concept. However, neither study incorporates Poisson population uncertainty.

3 Uncertainty in Coordination Games

Hoping to spark an interest in the Poisson population-uncertainty framework, we provide next an exposition of the innovative methodology in Ioannou and Makris (2019). Specifically, the authors investigate whether uncertainty about the number of stakeholders or idiosyncratic uncertainty about the profitability of the risky action may influence behavior, when these two types of uncertainty lead to a prediction of a unique equilibrium.[6] The experiments in Ioannou and Makris (2019) are single-shot. We thus proceed further to present novel results that pertain to repeated interactions in Poisson games.

3.1 Single-Shot Experiments

The objective in the study of Ioannou and Makris (2019) is two-fold: first, to investigate whether idiosyncratic uncertainty about economic fundamentals or uncertainty about the number of actual players impacts subjects' behavior and, second, to determine whether subjects' behavior is in line with the prediction of the respective theory. All experiments were conducted over the Internet. The Internet is ideal for Poisson population-uncertainty games as subjects cannot infer the number of participants, which is typically the case in a laboratory experiment.[7]

First, we describe the Poisson experiments. In the first stage, subjects were instructed that there would be a computer draw and that the number drawn would correspond to the number of subjects selected (i.e. activated) to participate in the next stage of the experiment. Subjects were explicitly told that the number drawn *would not* be revealed to them. The Poisson process was based on some mean $n = 17$. To circumvent the difficulties that would arise given the (assumed) unfamiliarity of many subjects with Poisson probabilities, the specific probabilities were applied onto a roulette wheel (see Figure 1). The authors instructed subjects the following.

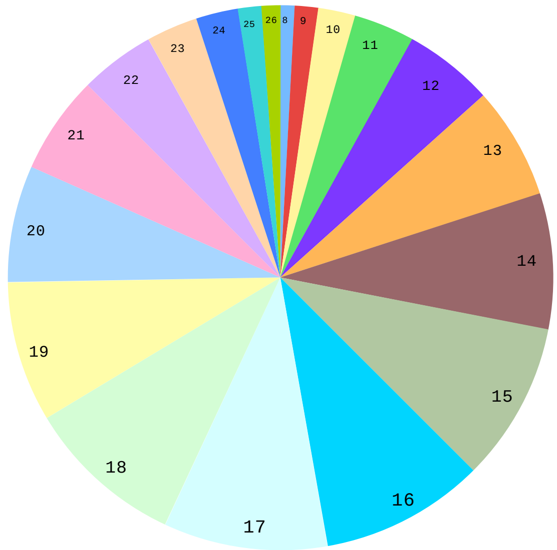
> You can see that the roulette is not a standard roulette; the number drawn can be any number between 8 and 26, but not all numbers are equally likely to be drawn. Numbers closer to 17 (the mean) are more likely to be drawn.

In the next stage, subjects had the option to invest in a risky project at an opportunity cost that would be profitable only if a sufficient number of subjects agreed to invest. However, in this stage, only *active* subjects could participate.

[6] A formal exposition of the theoretical predictions in Global games and Poisson games is provided in the Appendix.

[7] The added value of this approach is that it resembles how investors currently commit to their decisions; that is, after contemplating the pros and cons of various alternatives, investors will often place their (investment or short-selling) orders online.

Figure 1: Roulette Wheel for $n = 17$



Notes: Ioannou and Makris (2019) applied the specific Poisson probabilities onto a roulette wheel to give a pictorial sense of the likelihood of each possible number of active participants.

Analogous to the Poisson experiments, Global experiments also included a computer draw in the first stage. The drawn integer (between 5 and 95 inclusive) represented the state of fundamentals in the economy, where a higher number depicted a better state. Ioannou and Makris (2019) forewent indicating the actual integer drawn, yet provided subjects with a hint. The hint was an integer within a range of $+5$ and -5 from the number drawn. For example, if the number drawn was 25, subjects would receive a hint integer in the set of $\{20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30\}$, where each integer had the same probability of being drawn. Additionally, the number of subjects participating in the experiment was set at $N = 17$ and was common knowledge among participants. Similar to the Poisson experiments, in the second stage, subjects had the option to invest in a risky project at an opportunity cost that would be profitable only if a sufficient number of subjects agreed to invest.

In the controls, information about the number of players and economic fundamentals was common knowledge. The subjects were, then, asked to make a decision about investing in a risky project at an opportunity cost that would be profitable if the number of subjects choosing to invest was above the same threshold imposed in the Poisson and Global experiments.

Ioannou and Makris (2019) find that uncertainty about the number of actual players has a more significant impact on subjects' behavior than idiosyncratic uncertainty about economic fundamentals when focusing on parameters for which both Poisson and Global games predict a unique equilibrium. Crucially, subjects' behavior under Poisson population-size uncertainty is closer to the respective theoretical prediction than subjects' behavior under idiosyncratic uncertainty about economic fundamentals. In the context of speculative attacks, the theoretical prediction (see condition 3 in the Appendix) is that no attack will take place when the ratio of the short-selling cost per reward is greater than the probability of having sufficiently many players in the game. Consequently, we have immediately the following policy implication: increasing the size of the markets (by removing trade restrictions and market entry fees and/or improving cooperation between national financial networks) and imposing a Tobin tax on short-selling transactions may reduce the prior probability of an attack by currency speculators.

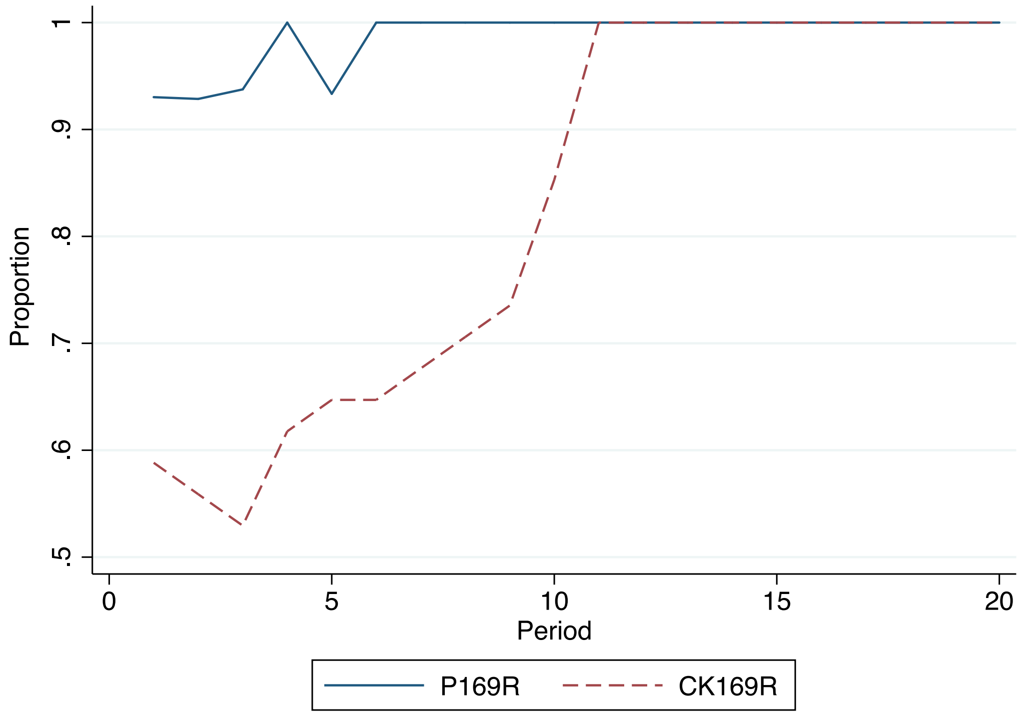
3.2 Repeated Experiments

In the study of Ioannou and Makris (2019), attention was restricted to single-shot experiments. In real life, however, for many applications of Coordination games, there are ample (personal or social) learning opportunities. Therefore, a natural question to ask next is how Poisson population uncertainty impacts subjects' behavior when interactions are repeated.[8] Echoing on the results of Cabrales, Nagel, and Armenter (2007) in the context of Global games, there might be variability in subjects' behavior in Poisson games when contrasting the short versus the longer term. We study next the impact of iterative play on subjects' behavior in the Poisson games and provide novel results. As a baseline, we implement a Coordination game with common knowledge in economic fundamentals and number of players.[9]

[8] We cannot conduct the same exercise for Global games in this set-up. The reason is the following. To simplify the cognitive environment faced by subjects, Ioannou and Makris (2019) focused on integer-based draws. However, the discrete grid can give rise to multiplicity of equilibria in Global games. Studying in a robust way the impact of learning on subjects' behavior in Global games would require maintaining a stable strategic environment across repeated interactions, while drawing the value of economic fundamentals at the start of each of the interactions. Unfortunately, such a stable strategic environment could not be guaranteed at the first stage because several possible values would have led to multiplicity of equilibria. Instead, we chose to focus on Poisson games and games with common knowledge in economic fundamentals and number of players to maintain clarity in our conclusions.

[9] The treatment and baseline are based on the parameters in Table 1 of Ioannou and Makris (2019) with acronyms P169 and CK169, respectively.

Figure 2: Subjects' Behavior with Repeated Interaction



Notes: The figure displays the proportion of subjects who chose not to undertake the risky project over the 20-period span in the treatment P169R and the baseline CK169R. The acronyms consist of the game type (*P* for the Poisson game or *CK* for the baseline game), the threshold that equals 16, the opportunity cost that equals 9, and letter "R" that stands for repeated interaction.

In Figure 2, we display the proportion of subjects who chose not to undertake the risky project (i.e. chose the 'safe' action) over the 20-period span in the experiments with repeated interaction. Based on the single-shot results in Ioannou and Makris (2019), where around 95% of the subjects chose not to undertake the risky project, we hypothesized that a very high percentage in the first period would be a significant deterrent to undertake the project in the next period (feedback is provided in each period) and so on and so forth. A proportion quite close to 1 was thus expected throughout the 20-period play. As shown in Figure 2, this prediction is confirmed. Through the first five periods in the Poisson games, the proportion of active subjects that chose not to undertake the risky project is over 90%. From the sixth period onwards, all active players chose not to undertake the project. It is important to reiterate that such behavior is also consistent with the respective theoretical prediction.

In the baseline, convergence to a proportion of 1 is slower. This is in line with earlier results documented in Brandts and Cooper (2006), and Cooper, Ioannou, and Qi (2018), where under fixed economic fundamentals and number of players (hence a stable strategic environment), some subjects *do* undertake the risky project in the early interactions; albeit, all choose the safe action after sufficiently many repeated interactions. Specifically, here, only from the eleventh period onwards the proportion of subjects who chose not to undertake the project equals 1. Before convergence, the proportion fluctuates between 0.53 and 0.85. The statistical analysis confirms that subjects' behavior in the earlier periods (i.e. before the eleventh period) is significantly different across the two games in each period.[10] In summary, the above results highlight that uncertainty regarding the number of actual players may be an important determinant of short and long-term empirical behavior in volatile environments with strategic complementarities.

4 Concluding Remarks

Coordination problems arise naturally and thus lie at the heart of many financial phenomena. The inherent multiplicity of equilibria led the literature to focus on the approach of Global games and Poisson games to resolve equilibrium indeterminacy. We provide an overview of the relevant literature, and suggest promising directions for future research where Poisson population uncertainty takes center stage. Hoping to encourage further such trajectories, we describe the methodology in Ioannou and Makris (2019) that implements population uncertainty and uncertainty about economic fundamentals in single-shot experiments, and provide novel results that pertain to iterative play. All in all, such research will amplify our knowledge of how to resolve Coordination problems, enrich policy debates and provide empirical explorations to refine theory.

[10] Indicatively, the p-values are 0.001, 0.006 and 0.087 in the first, fifth and tenth period, respectively.

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Appendix

Theoretical Predictions

We deploy the canonical Coordination game used in Ioannou and Makris (2019) in the context of startup investments under network externalities. Denote by N the number of players who decide whether to undertake the risky project or not.[11] Assume that indifferent players choose not to. Choosing not to undertake the risky project is the 'safe' action. Denote by T the opportunity cost, and Y the state of economic fundamentals with $Y \in \{Y_{min}, Y_{min}+1, ..., Y_{max}-1, Y_{max}\}$. Here, the state of economic fundamentals reflects profitability (i.e. the size of the reward gross of the opportunity cost). In particular, Y_{min} is the worst state in terms of profits, whereas Y_{max} is the most profitable state. The reward is provided if the number of investors is at least as high as $\alpha(Y)$. Therefore, after letting ν be the number of other investors, and $r \in \{0,1\}$ the player's decision to undertake the project ($r = 1$) or not, each player's payoff is given by

$$r\Big(\mathbf{1}_{\{\nu \geq \alpha(Y)-1\}} Y - T\Big).$$

The function $\alpha(.)$ and the opportunity cost are common knowledge. We set

$$\alpha(Y) = C - \frac{Y}{D}$$

with

$$C > 0, D > 0 \text{ and}$$

$$C - \frac{Y_{max}}{D} \leq 1.$$

The last condition states that in the state Y_{max}, the reward is provided even if only one player undertakes the risky project. Note that for $Y \geq \overline{Y} \equiv \alpha^{-1}(1)$, a single player undertaking the project is enough for the reward to be provided, while for $Y < \overline{Y}$ more than one investors will typically be required. We assume that $T < \overline{Y}$ to ensure that it is not weakly dominant to abstain from undertaking the project when $Y < \overline{Y}$. Let $\underline{Y} = max\{T, \alpha^{-1}(N)\}$. It is dominant to abstain for any state $Y < \underline{Y}$. We assume that $T > Y_{min}$ to ensure that this range of fundamentals is non-empty.

[11] In the lingo of the speculative attack model of Morris and Shin (1998), undertaking the risky project is analogous to short-selling (attacking) the currency.

We distinguish between three cases regarding agents' information about economic fundamentals and number of players. In the first case, there is common knowledge of economic fundamentals and number of players. In this game, the maximin outcome of no player undertaking the risky project is the unique equilibrium outcome for $Y < \underline{Y}$.[12] Furthermore, the payoff-dominant outcome of all N players investing is the unique equilibrium for $Y \geq \overline{Y}$. However, in the "grey area" (i.e. in the remaining area of economic fundamentals) there is multiplicity of equilibria. Depending on self-fulfilling beliefs both the maximin and payoff-dominant outcomes are equilibria. Figure 3 depicts the three regions (i.e. $Y < \underline{Y}$, $\underline{Y} \leq Y < \overline{Y}$, $\overline{Y} \leq Y$).

The other two cases are captured by the Global games and Poisson games. In Global games, the number of players in the game N is common knowledge and players receive private identically distributed and conditionally independent signals/hints about the unknown state of economic fundamentals Y. The set of signals is $\{x_{min}, x_{min}+1, ..., x_{max}-1, x_{max}\}$ and we denote a generic element of this set with x. In the Poisson games, economic fundamentals are common knowledge, whereas it is commonly understood that the number of actual players in the game is a Poisson random variable with mean n. In Poisson games, the only signal players receive reveals to them whether they are *active* players in the game.

Consider first the Global games. The unknown state Y is uniformly distributed and conditional on realized Y, x is uniformly distributed over $[Y - \varepsilon_Y, Y + \varepsilon_Y]$ with

$$2\varepsilon_Y < min\{Y_{max} - (C-1)D, max\{(C-N)D, T\} - Y_{min}\}.$$

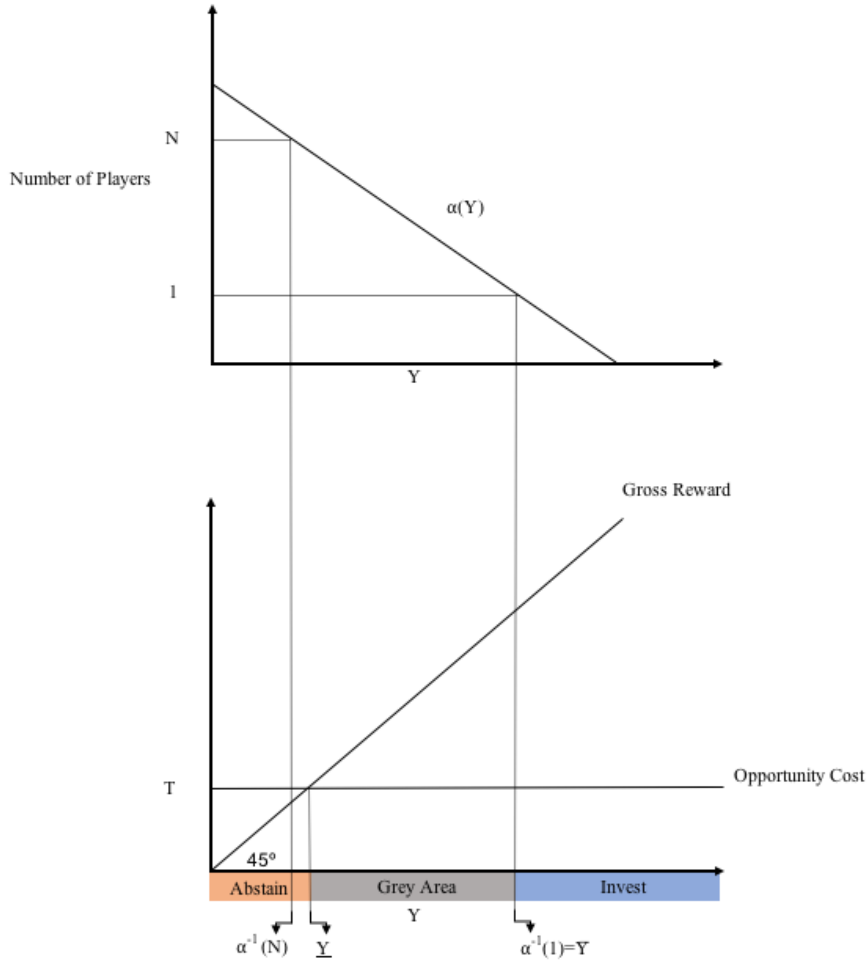
These distributions are common knowledge. In this case, there is a unique Bayesian Nash Equilibrium (BNE), where all players invest if and only if their signal is higher than x^*, where x^* is defined by

$$\frac{1}{2\varepsilon_Y} \int_{x^*-\varepsilon_Y}^{x^*+\varepsilon_Y} Y[1 - \sum_{j=0}^{\lceil a(Y)-2 \rceil} Bin(j, N-1, p(Y, x^*))]dY = T. \tag{1}$$

The symbolic function $\lceil \cdot \rceil$ rounds-up the fraction to the nearest integer from above, and $Bin(\cdot)$ is the binomial distribution where

[12] If $\underline{Y} = T$, then, investing when $Y = \underline{Y}$ is never profitable because even if enough players choose to undertake the risky project so that the reward is provided, the latter just covers the opportunity cost T. Therefore, it is weakly dominant to abstain from investing when $Y = \underline{Y} = T$.

Figure 3: The Three Regions



Notes: The orange region ($Y < \underline{Y}$) has the maximin outcome as an equilibrium. The blue region ($\overline{Y} \leq Y$) has the payoff-dominant outcome as an equilibrium. The grey area ($\underline{Y} \leq Y < \overline{Y}$) depicts the region where both the maximin and payoff-dominant outcomes are equilibria.

$$p(Y, x^*) = \frac{Y + \varepsilon_Y - x^*}{2\varepsilon_Y}.$$

To understand this condition, note first that receiving a signal x leads to the posterior that the state Y is uniformly distributed over $[x - \varepsilon_Y, x + \varepsilon_Y]$. Second, conditional on investing, the reward will be provided if at least $a(Y) - 1$ other agents also invest. Third, conditional on state being Y all signals lie in $[Y - \varepsilon_Y, Y + \varepsilon_Y]$. The term in the square brackets is therefore the probability that the reward is provided given that each and every of the other agents is expected to invest if and only if their signal is higher than x^*, and the left-hand side of the condition above is the associated expected benefit from investing when the received signal is x^*. The condition above simply says that an agent who has received the marginal signal x^* should be indifferent between investing or abstaining when all other agents are expected to invest if and only if their signal is higher than x^*.

However, the above result relies heavily on the assumption that the state and signal are continuous random variables. If, on the other hand, these are discrete random variables, then, there may not be a unique BNE. In fact, there may not even be a unique symmetric BNE in threshold strategies, where all agents undertake the risky project if and only if their signal is higher than a given threshold signal. If $Y \in \{Y_{min}, Y_{min}+1, Y_{min}+2, ..., Y_{max}-1, Y_{max}\}$ and $x_Y \in \{Y-\varepsilon_Y, Y-\varepsilon_Y+1, Y-\varepsilon_Y+2, ..., Y+\varepsilon_Y-1, Y+\varepsilon_Y\}$ where Y_{min}, Y_{max} and ε_Y are positive integers, then, a symmetric BNE in a threshold strategy with threshold x^* is given by the solution to the fixed-point problem

$$x^* \in \{k | Y_{min} - \varepsilon_Y - 1 \leq k \leq Y_{max} + \varepsilon_Y \text{ and}$$

$$U(x,k) \leq 0 \ \forall \ x \leq k \text{ and } U(x,k) \geq 0 \ \forall \ x > k\}, \tag{2}$$

where $U(x,k)$ is the expected payoff of an investor with signal x when all other players are expected to invest if and only if their signal is higher than k.

We turn next to the Poisson games. In a Poisson game, we assume that the number of players is a random variable drawn from a Poisson distribution with some mean n.[13] Given this parameter n, the probability that there are k players in the game is given by

$$p(k|n) = \frac{e^{-n}n^k}{k!} \quad \text{for } k = 0, 1, 2, ...$$

Let $F(\cdot \mid n)$ denote the Poisson cumulative distribution function with parameter n. It is straightforward to see that if $Y \leq T$, then, it is never profitable to invest.

[13] Consider the following example as a justification for this modelling choice. Suppose that the identity of every stakeholder is common knowledge and that binding individual orders for, say, investing in a startup must arrive by a given time. Standard theory suggests that each agent will decide on his action by taking the number of orders at the collector's disposal as given. However, the probability that a phone call to a busy switchboard will go through or the webpage of an online site will be uploaded successfully at times of high traffic decreases with the number of stakeholders. As a result, and under the assumption that the average number of successful phone calls or online visits is known, in a large environment, stakeholders should actually view the number of actual players in the Coordination game as a Poisson random variable.

Moreover, if $Y \geq \overline{Y}$, then, a single player undertaking the project is enough for the reward to be provided, and so every player finds it optimal to invest. However, for economic fundamentals within the remaining area, the unique symmetric equilibrium is the maximin outcome (where no player invests) if and only if

$$\Big[1 - F(\lceil \alpha(Y) \rceil - 2 \mid n)\Big] \leq \frac{T}{Y}. \tag{3}$$

Thus, in sharp contrast to the theoretical prediction in Global games that identifies a threshold strategy where all players invest as long as their signal is above some hurdle, here, nobody will invest in the risky project as long as the ratio of the opportunity cost per reward is greater or equal to the probability of having *sufficiently many* players in the game; otherwise, multiplicity of equilibrium outcomes still arises.